

Example: A charged particle and a time-independent \vec{B} -field, $\vec{B}(\vec{r})$.

$$\vec{F} = q \vec{v} \times \vec{B} = q \vec{v} \times (\vec{\nabla} \times \vec{A}); \quad F_x = q (\vec{v} \times (\vec{\nabla} \times \vec{A}))_x = q (v_y (\vec{\nabla} \times \vec{A})_z - v_z (\vec{\nabla} \times \vec{A})_y).$$

$$(\vec{v} \times (\vec{\nabla} \times \vec{A}))_x = v_y \frac{\partial}{\partial x} A_z - v_y \frac{\partial}{\partial y} A_x - v_z \frac{\partial}{\partial z} A_x + v_z \frac{\partial}{\partial x} A_y + v_x \frac{\partial}{\partial x} A_x - v_x \frac{\partial}{\partial z} A_x \quad (\pm)$$

$$= v_x \frac{\partial A_x}{\partial x} + v_y \frac{\partial A_y}{\partial x} + v_z \frac{\partial A_z}{\partial x} - v_x \frac{\partial A_x}{\partial z} - v_y \frac{\partial A_x}{\partial y} - v_z \frac{\partial A_x}{\partial z}.$$