Example: A charged particle and a time-independent $\vec{B}$-field, $\vec{B}(\vec{r})$.

$\vec{F} = q\,\vec{v}\times\vec{B} = q\,\vec{v}\times(\vec{\nabla}\times\vec{A})$; $F_x = q\left(\vec{v}\times(\vec{\nabla}\times\vec{A})\right)_x = q\left(v_y(\vec{\nabla}\times\vec{A})_z - v_z(\vec{\nabla}\times\vec{A})_y\right)$.

$$\left(\vec{v}\times(\vec{\nabla}\times\vec{A})\right)_x = \overset{(2)}{v_y\frac{\partial}{\partial x}A_y} - \overset{(5)}{v_y\frac{\partial}{\partial y}A_x} - \overset{(6)}{v_z\frac{\partial}{\partial z}A_x} + \overset{(3)}{v_z\frac{\partial}{\partial x}A_z} + \overset{(1)}{v_x\frac{\partial}{\partial x}A_x} - \overset{(4)}{v_x\frac{\partial}{\partial x}A_x} \quad (\pm)$$

$$= \overset{(1)}{v_x\frac{\partial A_x}{\partial x}} + \overset{(2)}{v_y\frac{\partial A_y}{\partial x}} + \overset{(3)}{v_z\frac{\partial A_z}{\partial x}} - \overset{(4)}{v_x\frac{\partial}{\partial x}A_x} - \overset{(5)}{v_y\frac{\partial}{\partial y}A_x} - \overset{(6)}{v_z\frac{\partial}{\partial z}A_x}.$$

$$= \frac{\partial}{\partial x}(\vec{v}\cdot\vec{A}) - \frac{dA_x}{dt}.$$

(q, $\dot{q}$ are independent, $\frac{\partial \dot{q}}{\partial q} = 0$.)

$$\frac{dA_x}{dt} = \frac{\partial A_x}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial A_x}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial A_x}{\partial z}\frac{\partial z}{\partial t} + \underbrace{\frac{\partial A_x}{\partial t}}_{=0}.$$

But $A_x = \frac{\partial}{\partial v_x}(\vec{v}\cdot\vec{A}) = \frac{\partial}{\partial v_x}(v_x A_x + v_y A_y + v_z A_z)$.

Therefore $F_x = q\,\vec{v}\times\vec{B} = q\left[\frac{\partial}{\partial x}(\vec{v}\cdot\vec{A}) - \frac{d}{dt}\frac{\partial(\vec{v}\cdot\vec{A})}{\partial v_x}\right] = -\frac{\partial U}{\partial x} + \frac{d}{dt}\frac{\partial U}{\partial v_x}$,

with $U = -q\,\vec{v}\cdot\vec{A}$.

$L = \frac{1}{2}m v^2 + q\,\vec{v}\cdot\vec{A}$.