August 2023 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $q_e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\,400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\,7685 * 10^{-3}$ m K
Planck radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\lambda)) - 1]^{-1}$

Useful integral:

$\int \sin^3 x\, dx = -(1/3) \cos x (\sin^2 x + 2)$

Section I:

Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

Problem 1:

On the basis of hydrostatic equilibrium, one can make an estimate that the pressure at the center of the Sun is $5*10^{14}$ N/m^2.

(a) If the Sun is purely hydrogen (with a number density of $3*10^{30}$/m^3) and can be treated as an ideal gas, what is the temperature at the solar center?

(b) What is the typical velocity of hydrogen ions in the solar center?

Problem 2:

Consider an atomic nucleus.

(a) Why do nuclei tend to have approximately the same number of neutrons and protons?

(b) As you increase the element atomic number, why does the ratio of neutrons to protons increase?

(c) Why do light nuclei release energy via fusion while heavy nuclei release energy via fission?

Problem 3:

A truck is traveling on a level road. The driver suddenly applies the brakes, causing the truck to decelerate by an amount 3m/s^2. There is a 90 kg box in the back of the truck. The coefficient of sliding friction between the truck and the box is 0.2. Find the acceleration of the box relative to

(a) the truck and

(b) the road.

Problem 4:

A particle in some unspecified potential is in the initial normalized state

$\psi(x) = a\, x\, e^{-x/b}$, $x \geq 0$,

$\psi(x) = 0$, $x < 0$.

(a) Where are you most likely to find the particle if you were to measure its position?

(b) What is the probability of finding the particle in the region $x > 0$?

Problem 5:

A car of mass m is slowed down by a drag force $F = -kv^2$. How far will the car travel before its speed is halved?

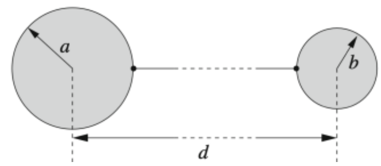
Problem 6:

Two conducting spheres of radii a and b < a, respectively, are connected by a thin metal wire of negligible capacitance. The centers of the two spheres are at a distance d >> a > b from each other. A total net charge Q is located on the system. Evaluate to zeroth order approximation, neglecting the induction effects on the surfaces of the two spheres,

(a) how the charge Q is partitioned between the two spheres,

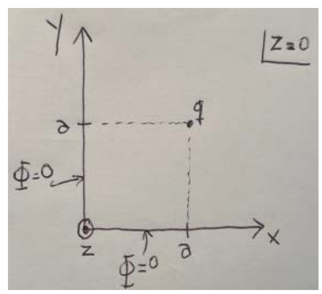
(b) the value V of the electrostatic potential of the system (assuming zero potential at infinity) and the capacitance C = Q/V,

(c) the electric field at the surface of each sphere, comparing the magnitudes and discussing the limit b --> 0.



Problem 7:

Consider a conducting (i.e. metallic) region defined by two half-planes connected to the ground (i.e. at zero potential Φ), as shown in the figure. These planes are perpendicular to one another i.e. one is defined by x = 0 and y > 0, and the other by y = 0 and x > 0. A charge q is located are position P = (a, a, 0) as shown. The region where the charge resides is in the vacuum, and it is called here the first quadrant. Note that this is a 3D problem, not just 2D.



(a) Using the method of images find the scalar potential Φ(r) in the first quadrant at an arbitrary point r = (x, y, z).

(b) Verify that the potential you found in (a) cancels in the two half-planes.

(c) Find the force on the charge q induced by the planes.

Problem 8:

The atomic mass of $^{30}_{15}P$ is 29.978307 u, while the atomic mass of $^{30}_{14}Si$ is 29.973770 u. Which decays to which and by what process? (Hint: Don't forget to consider the net charge of the full atoms.)

Problem 9:

Consider an electron-positron collider, a circular ring with a beam of electrons and a beam of positrons traveling in opposite directions. The mass of the electron is 0.5 MeV/c^2.

(a) If you wanted to increase the speed of the particles in the beams (accelerate them in the direction of their velocity) could you use electric fields, magnetic fields, or either? Why?

(b) If each beam is accelerated to an energy of 100 GeV, and the ring has a circumference of 1 km, how long would a trip around the ring take in the rest frame of the electron?

Problem 10:

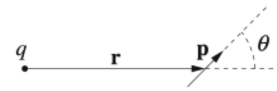
Neptune orbits the Sun ($M_{sun} = 1.99*10^{30}$ kg) with an average orbital radius of r = 30.1 Astronomical Units = $4.514*10^{12}$ m, and a typical velocity of 5.42 km/s. What velocity would be required for Neptune to leave the solar system from its current radius?

Section II:

Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:

An electric dipole **p** is located at a distance **r** from a point charge q, as shown in the figure. The angle between **p** and **r** is θ. Evaluate the electrostatic force on the dipole.



Problem 12:

Suppose you have two non-interacting, spin-less particles in a one-dimensional potential, each in one of two single-particle, orthonormal eigenstates $\psi_1(x)$ and $\psi_2(x)$ of that potential. Now suppose you measure the position of one of the particles in an ensemble of identically prepared experiments. What is the expectation value of position if the particles are

(a) distinguishable particles,

(b) identical bosons,

(c) identical fermions?

In any one of these cases, if there is more than one answer to the question, provide all of them.

Problem 13:

For this one-dimensional problem let the x-axis point downward. A vertical spring has a spring constant k = 48 N/m. At t = 0 a force F(t) = (51 N)sin(4t) ($t \geq 0$ with t in seconds) is applied to a 30 N weight which hangs in equilibrium at the end of the spring. Neglecting damping, find the position of the weight at any time t.

Problem 14:

The volume between two concentric spherical surfaces of radii a and b ($a < b$) is filled with an inhomogeneous dielectric with permittivity $\varepsilon = \varepsilon_0/(1 + \kappa r)$, where ε_0 and κ are constants and r is the radial coordinate. Thus **D**(r) = ε**E**(r). A charge Q is placed on the inner surface, while the outer surface is grounded. Find:

(a) The displacement **D**(r) and the field **E**(r) in the region $a < r < b$.

(b) The capacitance of the device.

(c) The volume polarization charge density in the region $a < r < b$.

(d) The surface polarization charge density at r = a and at r = b.

Problem 15:

A sphere has a mass m with uniform density ρ and radius a. Calculate the moment of inertia about an axis through its center. Use spherical coordinates and do not just write down the answer, show your calculation.