August 2024 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $q_e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\,400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\,7685 * 10^{-3}$ m K
Planck radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\lambda)) - 1]^{-1}$

Useful integral:

$\int x \sin^2 x\, dx = x^2/4 - x \sin(2x)/4 - \cos(2x)/8$

Section I:

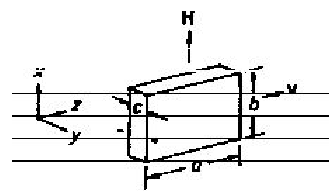
Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

Problem 1:

A photon rocket emits a beam of light instead of the hot gas of an ordinary rocket. How powerful a light source would be needed for a photon rocket with thrust equal to that of a space shuttle (35 MN)? Compare the power of this photon rocket with humanity's total electric power-generating capability, about 10 TW.

Problem 2:

An uncharged metal block has the form of a rectangular parallelepiped with sides a, b, c. The block moves with velocity v in a magnetic field of intensity **H** as shown in the figure. What is the electric field intensity in the block and what is the electric charge density in and on the block?



Problem 3:

A uniform rope of length L and mass M is coiled on the table. You lift one end of the rope with acceleration a. Find the force of the rope held by your hand when a length y of the rope is above the table.

Problem 4:

(a) A telescope with a primary mirror with a diameter of 20 m records 314 photons/second from a star. Assume that no photons are lost in our atmosphere and that the telescope plus detector system is 1% efficient. What is the flux of photons/s/m^2?

(b) What is the energy flux in W/m^2 if the effective wavelength is 500 nm?

(c) You observe this star for a year and see that it inscribes a small circle with diameter 1/3600 degrees relative to the fainter and much more distant stars. The circular motion repeats with exactly one year period. What is the distance to this star? The Earth is 150 million km from the Sun.

(d) How much energy does this star emit in all directions per second into the waveband that was detected (in part a)?

Problem 5:

Earth's magnetic field ends abruptly on the sunward side at approximately the point where the magnetic energy density has dropped to the same value as the kinetic energy density in the solar wind. Near Earth, the solar wind contains about 5 protons and 5 electrons per cm^3 and flows at 400 km/s. Treating Earth's magnetic field as that of a dipole with dipole moment $8*10^{22}$ J/T, estimate the distance to the point above the equator where the field ends.

Problem 6:

A research reactor produces fluxes of neutrons to undertake measurements of the structure of materials. The neutrons are emitted from an aperture in the reactor wall. The beam contains a wide mixture of neutrons with different energies. Bragg reflections by a crystal can be used to separate the a monoenergetic beam from the mixture in a similar way to how a diffraction grating separates colors of light into a monochromatic beam.

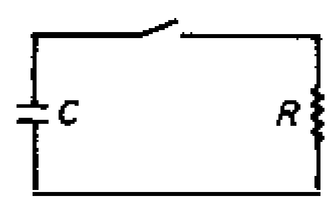
(a) Suppose that Beryllium crystals with a Bragg plane spacing d = 1.80 Angstroms ($1.8*10^{-10}$m) are used to separate neutrons with an energy of 0.10 eV. What is the de Broglie wavelength λ in Angstroms of a neutron with an energy of 0.010 eV? Note that the neutrons are nonrelativistic.

(b) Calculate the angle 2θ between the incident beam and scattered beam directions for this wavelength of neutrons based on the Bragg condition that the path difference for scattered neutrons is λ between successive crystal planes.

Problem 7:

(a) The capacitor in the circuit in the figure is made from two flat square metal plates of length L on a side and separated by a distance d. What is the capacitance?

(b) Show that if any electrical energy is stored in C, it is entirely dissipated in R, after the switch is closed.



Problem 8:

A rebellion ship moving at v = 0.999c flies past a stationary star destroyer. The star destroyer fires a laser canon with a very short energy pulse of 105.3 μJ, such that it strikes the rebellion ship at the point of closest approach. What is the energy of the laser pulse according to the rebellion ship?

Problem 9:

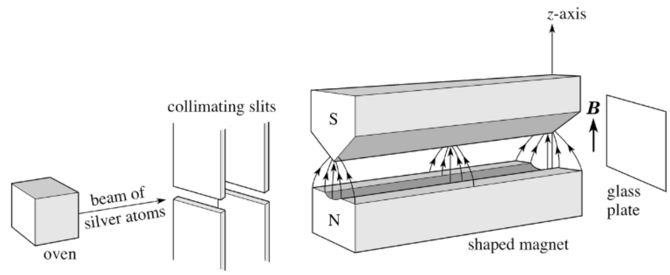
The sketch below illustrates the setup of the Stern-Gerlach experiment that historically confirmed the quantum nature of an atomic-scale system, specifically, the quantization of angular momentum. In this setup, silver atoms are heated in an oven, creating a beam of atoms that goes through a collimator. The beam is then subjected to an inhomogeneous magnetic field before colliding with a glass plate.

Answer the following questions about this experiment.

(a) Why is an inhomogeneous magnetic field needed?

(b) If the electron spin angular momentum is a classical quantity, what will be the distribution of silver atoms on the plate?

(c) Based on your knowledge of electron spin, i.e., the quantum mechanical description, how do you predict the distribution of silver atoms on the plate?



Problem 10:

At $t < 0$, a particle with mass m is located in a one-dimensional potential described by

$U(x) = \infty$, $x < 0$, $x > L$,

$U(x) = 0$, $0 \le x \le L$.

A potential of $\Delta V(x) = \alpha x$ is added to $0 \le x \le L$ from $t = 0$ with $\alpha << 1$.

(a) What are the energy eigenvalues and eigenstates for the system at $t < 0$?

(b) Calculate the first order correction to the lowest two energy eigenvalues at $t \rightarrow \infty$ by treating $\Delta V(x)$ as a perturbation.

Section II:
Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:

A spin ½ particle is represented by the following spinor: $\chi = A\begin{pmatrix} 1-2i \\ 2 \end{pmatrix}$. Here, we used the eigenspinors (eigenvectors) of the operator $\hat{S}_z$ as our basis; A is a normalization constant.

(a) If you measure $\hat{S}_z$ of this particle, what values do you get and what is the probability of each.

(b) Find $\langle \hat{S}_z \rangle$.

(c) If you measured $\hat{S}_x$ instead, what values do you get and what is the probability of each?

Hint: $\chi^{+}_{(x)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\quad \chi^{-}_{(x)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ in the basis $\hat{S}_z$

Problem 12:

(a) Find any equilibrium points for a Higgs-like potential of the form $V(x) = -50x^2 + x^4$.

(b) Taylor expand the potential around a minimum and demonstrate that sufficiently near the minimum, this system can be treated as a harmonic oscillator.

(c) How far from the minimum do you need to be such that the harmonic oscillator term is not the largest term of the expansion?

Problem 13:

(a) When the gas isothermally expands against a fixed external pressure, would the expansion be reversible or irreversible? Explain your answer.

(b) Will the entropy of the ideal gas increase, decrease of remain fixed during the expansion? Explain your answer.

(c) Suppose the external pressure is zero, what would be the entropy change of the gas, its surroundings, and for the universe? If appropriate, express your answer in terms of temperature and volume.

Problem 14:

Use the Euler-Lagrange equation to show that the shortest path between two points on a plane is a straight line.

Problem 15:

Consider an ideal gas of N atoms following classical statistics. Find the contribution to the average Helmholtz free energy and entropy from the internal electronic level structure considering the atoms as two-level systems with the ground state energy 0 and excited state energy E. The system's temperature is T. The internal electronic energy levels are non-degenerate.

The Helmholtz free energy is defined as $F = U - TS$. $F = -Nk_BT \ln(Z)$.