August 2025 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $q_e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\ 400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\ 7685 * 10^{-3}$ m K
Planck radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\lambda)) - 1]^{-1}$

Useful integral:

$\int \sin(mx) \sin(nx)\, dx = \sin((m - n)x)/(2(m - n)) - \sin((m + n)x)/(2(m+ n))$.

Section I:

Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

Problem 1:

The cutoff wavelength for the photoelectric effect in tungsten metal is 274 nm.

(a) What is the stopping potential for the photoelectron when the system is illuminated with 200 nm light?

(b) What is the De Broglie wavelength of the photoelectron when it is ejected from the material under these conditions?

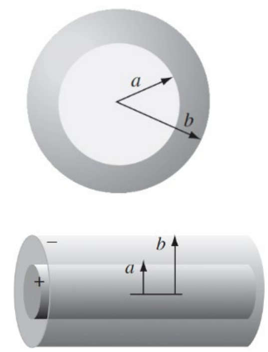
Problem 2:

A long coaxial cable carries a uniform volume charge density ρ on the inner cylinder (radius a), and a uniform surface charge density on the outer cylindrical shell (radius b). This surface charge is negative and is of just the right magnitude that the cable as a whole is electrically neutral.

(a) Find the electric field inside the inner cylinder ($r < a$).

(b) Find the electric field between the cylinders ($a < r < b$).

(c) Find the electric field outside the cable ($r > b$).



Problem 3:

(a) Sketch the energy level diagram of all possible (n,l) states of the hydrogen atom up to n = 3. (Ignore spin.)

(b) How many transitions between the (n,l) states in part (a) are allowed?

Draw all possible transition in the diagram you have in part (a).

(c) Which transition in part (b) produces the largest energy photon?

Calculate the energy of that photon.

Problem 4:

A given spring is “lousy” in the sense that it loses its elastic properties over time. To describe this, let the restoring force of the spring be expressed as $F(x) = -k(x - x_0)e^{-t/\tau}$, where k and τ are constants, x is the position of the mass m attached to the spring, and x_0 is the position of equilibrium.

(a) Does the potential energy exist? If so, find it. If not, motivate your answer.

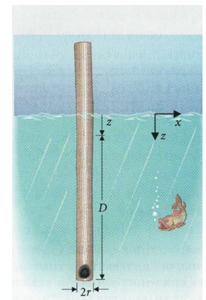
(b) Is mechanical energy conserved? If so, prove it. If not, determine the amount of energy lost per unit time.

Problem 5:

Consider a particle in a 1D box of length L, i.e. there are hard walls at x = 0 and at x = L. Assume that the particle is in the ground state. At t = 0 the wall at x = L is suddenly moved to x = 2L. What is the likelihood to find the particle in the ground state of the larger box?

Problem 6:
Protons with kinetic energy of 1 GeV in a narrow beam are diffracted by Oxygen nuclei. The first minimum is observed at an angle of 8.5 degree. Calculate the radius of the Oxygen nuclei.

Problem 7:
A log of length L and radius r is weighted at one end with a lump of lead so it can float vertically in water (of density ρ_{water}). The total mass is M. The log is pushed in the water by a small amount compared to the equilibrium position.

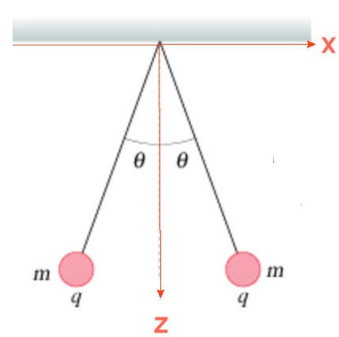


(a) Find the equation of motion for the log and describe the motion.
(b) If the motion is periodic, give an expression for the period.

Problem 8:
Consider that the Universe is spherical and an adiabatic expansion of a mono-atomic hydrogen gas (γ = 5/3). The current Universe's radius is at least 46 billion light years, and its temperature is 2.7 K. Estimate the temperature when the Universe had the size of a cherry (1 cm radius).

Problem 9:
Two identical bodies with temperature-independent heat capacities C_0 are initially at different temperatures T_1 and T_2. A Carnot cycle is run between them (with infinitesimal steps) until they have a common temperature T_F. What is the value of T_F?

Problem 10:
Two small metallic spheres are suspended from a common point as pendulums by light strings of length 0.3 m. The mass of each sphere is 0.2 g. They are given the same electric charge. They come to equilibrium when each string is at an angle of 5 degrees to the vertical. What is the magnitude of the charge on each sphere?



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Section II:

Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:

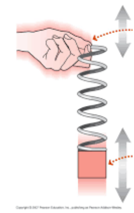
Consider a parallel-plate capacitor with circular plates of radius R and separation $d \ll R$.
The capacitor is being charged by a time-dependent current $I(t) = I_0 \sin(\omega t)$.

(a) Use Ampere's law to compute the displacement current density between the plates as a function of radius r (for $r < R$).

(b) Find the magnetic field $\mathbf{B}(\mathbf{r}, t)$ inside the capacitor (between the plates).

(c) Discuss the behavior of $\mathbf{B}(\mathbf{r}, t)$ for $r > R$, assuming the current is confined within the radius R.

Problem 12:



A block with mass m is hanging on a spring with spring constant k.
The spring is held on the other side by a hand.

(a) If the hand moves as $y_h(t) = A\sin(\omega t)$, what is the equation of motion of the block? Assume that the spring is ideal and there is no damping. Assume that ω is not the natural frequency of the spring.

(b) What "initial conditions" at $t = 0$ are needed so that the block oscillates with a single frequency ω? Under those conditions, what is the oscillation amplitude of the block? What is the oscillation phase of the block relative to the hand?

Problem 13:

The Doppler effect is the change in the frequency of a wave caused by a moving source or observer.
Let's assume a source produces sound wave with the speed v and frequency f_0.

(a) When the source moves toward a stationary observer with speed v_s, what is the frequency of the sound heard by the observer?

(b) When the observer moves toward the stationary source with speed v_o, what is the frequency of the sound heard by observer?

(c) For a light wave with frequency f and speed c, show that the classical Doppler effect violates special relativity. Use special relativity to derive the frequency of the light wave when the source is moving toward observer with the speed v_s.

Problem 14:

A system is governed by the Hamiltonian $H = \begin{pmatrix} -E_0 & V \\ V & E_0 \end{pmatrix}$.

(a) When the energy is measured, what are the possible outcomes?
(b) At time t = 0 the system is in the state $\binom{1}{0}$. What is the energy expectation value at time t = 0 and what is the energy expectation value at subsequent times?
(c) Now let $E_0 = 3V/4$. What is the state of the system at time t?
(d) What is the probability to find the system in the state $\binom{0}{1}$ at time t?

Problem 15:

(a) Find the magnetic field at a distance s from the long straight wire, carrying a steady current I.
(b) Find the flux of **B** through a square loop of wire (side a) that lies on a table, a distance s from a very long straight wire with a current I, as shown in the figure.
(c) If someone now pulls the loop directly away from the wire, at speed v, what emf is generated? In what direction (clockwise or counterclockwise) does the current flow?