August 2025 Qualifying Exam

Part II

Mathematical tables are allowed. Formula sheets are provided.

Calculators are allowed.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $q_e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\,400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\,7685 * 10^{-3}$ m K
Planck radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\,\lambda)) - 1]^{-1}$

Solve 6 out of the 8 problems! (All problems carry the same weight)

Problem 1:

Follow these steps to estimate the total energy of a helium atom.

(a) What would be the total energy of a helium atom in its ground state in the approximation that you ignore completely the electrostatic force between the two electrons?

(b) Interpret the sign of your answer in part (a). What does it mean for the stability of the He atom? How good of an estimate do you think this? Does it over or underestimate the energy? Why?

(c) Now consider the correction to the potential energy due to the Coulomb interaction between the electrons. Assume that the electrons are classical particles in the first Bohr orbit (but remember $Z = 2$ for the helium nucleus). The two electrons will always stay on opposite sides of the orbit from each other to minimize their energy. What is the potential energy due to their interaction under this assumption? Combine this number with your answer to part (a) to obtain the total energy of the two electrons. Compare your result to the observed value of -79.0 eV.

Problem 2:

A thin ring-shaped wire of mass M carries uniform electrical charge Q. The wire rotates about its center axis with rotational velocity $\boldsymbol{\omega}$.

(a) What is the ratio of its magnetic dipole moment to its angular momentum?

(b) Classically, the electron is considered a spinning charged sphere. According to quantum mechanics, the angular momentum of the spinning electron is $h/4\pi$, with h the Planck constant. What torque acts on the electron at an angle θ between the magnetic moment and an outer magnetic field **B** in terms of the ratio derived in (a)? Explain how (a) applies.

(c) Derive the semi-classical precession angular velocity for an electron in a 3 Tesla magnetic field.

Problem 3:

A particle of mass m in three dimensions is subjected to the radial potential $V(r) = \frac{\hbar^2 \kappa}{2m} \delta(r - R)$.

Here $R > 0$ and $\kappa > 0$ are parameters. In what follows, we consider s-waves only.

(a) Compute the wave functions that are positive-energy solutions.

Hint: Parametrize the outside wave function such that its asymptotic (i.e. for $r >> R$) form is $\psi(r) = \frac{\sin(kr+\delta(k))}{r}$ and its energy is $E = \hbar^2 k^2/(2m)$. Derive a formula for the phase shift $\delta(k)$.

Hint: Use $\cot(\alpha + \beta) = \frac{\cot\alpha \cot\beta - 1}{\cot\alpha + \cot\beta}$.

(b) Take the limit $\kappa \rightarrow \infty$ and compute the phase shift. What physical system is this?

Hint: Look also at the inside (i.e. for $r < R$) wave function.

(c) Assume and attractive potential, i.e. $\kappa < 0$, and make an ansatz for a bound-state wave function with negative energy $E = -\hbar^2\gamma^2/(2m)$. Under what conditions (on R and κ) will the system have a bound state?

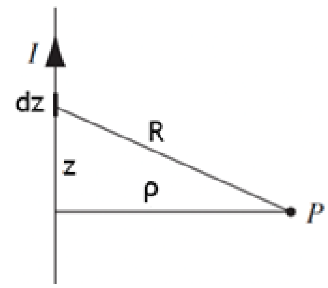
Problem 4:

An infinite straight wire carries the current

$$I(t) = \begin{cases} 0, \text{for } t \le 0 \\ I_0, \text{for } t \ge 0 \end{cases}.$$

That is, a constant current I_0 is turned on abruptly at t = 0.

Find the scalar and vector potentials in the Lorentz gauge.

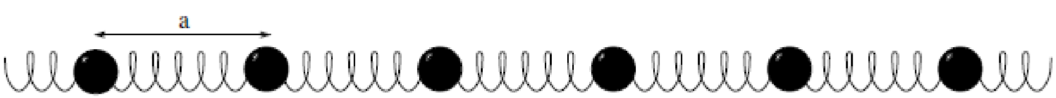


Problem 5:

Consider the N equally spaced, identical balls connected by springs. All balls have mass m, all springs have spring constant β, and adjacent balls are separated by distance a at equilibrium.

(a) Write down the equation of motion for each ball. You can assume periodic boundary conditions.

(b) Longitudinal wave can propagate along the springs and balls. Prove that the dispersion relation between the wave angular frequency ω and wavevector k is given by $\omega = 2\sqrt{\frac{\beta}{m}}\left|\sin\left(\frac{ka}{2}\right)\right|$.



Problem 6:

Consider a rectangular lamina with dimensions A and B, and mass M. The lamina is attached to a wall with a nail placed at a point P = P(x,y), where the coordinates x and y are referred to the center of the lamina.

(a) Determine the position of stable equilibrium.

(b) Describe the motion of the lamina when it is slightly displaced from this equilibrium position.

Problem 7:

A point electric dipole located at the origin has a time-dependent dipole moment given by

$\mathbf{p}(t) = p_0 \cos(\omega t)\, \mathbf{k}$.

In the far field region this gives

$\mathbf{E}(\mathbf{r}, t) = \mu_0 p_0 \omega^2/(4\pi r) \sin\theta \cos(\omega t - kr)\, \mathbf{e}_\theta$,

$\mathbf{B}(\mathbf{r}, t) = \mu_0 p_0 \omega^2/(4\pi rc) \sin\theta \cos(\omega t - kr)\, \mathbf{e}_\phi$,

for electric and magnetic field.

(a) Calculate the time-averaged power radiated per unit solid angle $dP/d\Omega$.

(b) Integrate over all angles to find the total radiated power.

(c) Discuss the angular distribution of the radiation and where the radiation is strongest.

Problem 8:

A horizontal one-meter-long stick is moving with velocity u **j** along the y−direction in the lab frame S. Reference frame S' is moving with velocity $\mathbf{v} = v\,\mathbf{i}$ along the x-direction relative to S.

(a) Using the Lorentz transformation, derive the velocity **u**' of the stick measured by an observer at rest in S' in terms of u, v, and c.

(b) In the S frame the stick is oriented horizontally. Assume that the stick is crossing the x-axis at t = 0, with its left and right ends at $x_L = -0.5$ m, $y_L = 0$ and $x_R = +0.5$ m, $y_R = 0$ respectively.
The rod is not horizontal for observers in the S' frame. Find the vertical distance that the rod travels while crossing the x'-axis according to an observer at rest in S', in terms of u, v and c.

(c) Find the length of the stick and the angle θ that the stick makes with the x'-axis in terms of u, v, and c according to the observer in the S' frame.