January 2025 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $q_e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\ 400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\ 7685 * 10^{-3}$ m K
Planck radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\,\lambda)) - 1]^{-1}$

Useful integral: $\int \sin^2(x)dx = \frac{1}{2} x - \frac{1}{4} \sin(2x)$

Hund's Rule

- The level with the largest multiplicity has the lowest energy.
- For a given multiplicity, the level with the largest value of L has the lowest energy.
- For less than half-filled shells: The component with the smallest value of J has the lowest
- For more than half-filled shells: The component with the largest value of J has the lowest energy.
- When the number of electrons is 2l + 1, i.e. when the shell is half filled, there is no multiplet splitting.

Section I:

Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

Problem 1:

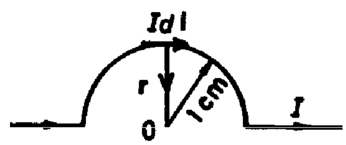
An electron is in a state with $l = 2$ in an atom.

(a) What is the magnitude of $\boldsymbol{L}$?

(b) What are the allowed quantum numbers j, and what is the magnitude of the vector $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$?

Problem 2:

As in the figure, an infinitely long wire carries a current I = 1.0 A. It is bent so as to have a semi-circular detour around the origin, with radius 1.0 cm. Calculate the magnetic field **B** at the origin.



Problem 3:

A radio telescope consists of two antennas separated by a distance of 200 m. Both antennas are tuned to a particular frequency, such as 20 MHz. The signals from each antenna are fed into a common amplifier, but one signal first passes through a phase adjuster that delays its phase by a chosen amount so that the telescope can look in different directions. When the phase delay is zero, plane radio waves that are incident vertically on the antennas produce signals that add constructively at the amplifier. What should the phase delay be so that signals coming from an angle $\theta = 10^{\circ}$ with the vertical (in the plane formed by the vertical and the line joining the antennas) will add constructively at the amplifier?

Problem 4:

Two supernovae explode with coordinates

$(ct_1, x_1, y_1, z_1) = (0, 1, 0, 0)$ and $(ct_2, x_2, y_2, z_2,) = (3, 1, 5, 0)$, respectively. (Distances are measured in arbitrary units.)

(a) Does a reference frame exist where these two supernovae occur simultaneously?

What about a frame where they occur at the same spatial location?

(b) If applicable, in the frame where the two events occur at the same time, what is the distance between them?

If applicable, in the frame where the two events occur at the same location, what is the time difference between them?

Problem 5:

A particle with mass m and with zero energy has a time-independent wave function $\psi(x) = x\ \exp(-x^2/L^2)$. Determine the potential energy U(x) of the particle.

Problem 6:

In a deuterium-tritium (D—T) nuclear reaction, 17.6 MeV of energy is released. You are given a laser-fusion fuel pellet of mass 5 mg which is composed of equal parts (by mass) of deuterium and tritium.

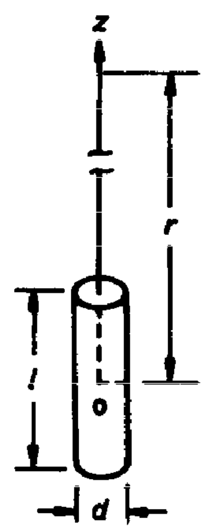
(a) If half the deuterium and an equal number of tritium nuclei participate in D—T fusion, how much total energy (in Joules) is released?

(b) At what rate must pellets be fueled in a power plant with 3 GW thermal power output?

(c) What amount of fuel would be needed to run the plant for 1 year? Compare with the 3.6 megatons of coal needed to fuel a comparable coal-burning power plant.

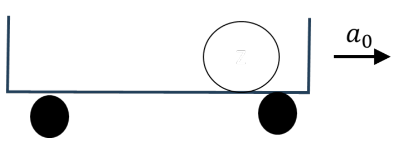
Problem 7:

A cylindrical conducting rod of diameter *d* and length *l* (*l* >> *d*) is uniformly charged in vacuum such that the electric field near its surface and far from its ends is E_0. What is the electric field at *r* >> *l* on the axis of the cylinder?



Problem 8:

A cylinder with mass m, radius R, and uniform density starts rolling without slipping from rest in a train that accelerates with constant acceleration a_0. The axis of the cylinder is perpendicular to the motion of the train as shown below. Find the acceleration of the cylinder in the lab frame (rest frame) and in the train frame.



Problem 9:

A conducting loop of area A and resistance R lies perpendicular to a uniform magnetic field **B**. The loop is then rotated at a uniform rate until it is upside down in time t. Find the work done in flipping the loop.

Problem 10:

How much work does one mole of monoatomic ideal gas do during adiabatic expansion when its volume increases by a factor of 2?

The gas is initially at room temperature T = 294 K and has three translational degrees of freedom. Consider the expansion to be quasistatic. Calculate your answer in Joules. How much does the entropy of the gas change? The ideal gas constant is R = 8.31 J/(K·mol).

Section II:

Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:

Studies of the origin of the solar system suggest that sufficiently small particles might be blown out of the solar system by the force of sunlight. To see how small such particles must be, compare the force of sunlight with the force of gravity, and find the particle radius r at which the two are equal. Assume that the particles are spherical, act like perfect mirrors, and have a density 2 g/cm^3.

The solar luminosity is $L = 3.83*10^{26}$ Watts and the solar mass is $M_s = 1.99*10^{30}$ kg.

Why do you not need to worry about the distance from the Sun?

Problem 12:

Titanium (Ti) is element No. 22 in the Periodic Table.

(a) Write down the electronic shell configuration of Ti (i.e., $1s^2$, 2s….2p….).

(b) Find the total orbital angular momentum L, total spin angular momentum S, and the total angular momentum quantum number J for the ground state of the Ti atom.

(c) Write down the corresponding spectroscopic term symbol $^{2S+1}X_J$ for the ground state, where X represents the orbital angular momentum.

(d) In a photoemission experiment, an electron is ejected from the 2*p* shell. In this process, we will assume that the angular momentum states of the valence electrons remain frozen.

Find the possible values of $S, L, \text{and } J$ for the ionized 2p level, ignoring all other levels in the atom.

(e) Couple the ground state orbital angular momentum found in (b) with the one you found in (d). Do the same for the spin angular momenta in (b) and (d). What possible values of L and S do you get?

(f) Find the six possible spectroscopic term symbols for the ionized Ti atom (or final state of the photoemission process). Just leave the value of J blank, i.e., write them as ^{2S+1}X

(g) Find the corresponding multiplicities.

Problem 13:

The potential energy of a mass m as a function of position x is given by $U(x) = ax^2 + bx + c$, with a, b, c positive constants.

(a) Find the equilibrium position of the mass.

(b) If the mass is released from rest at x = 0 at t = 0, find its position as a function of time assuming no resistive force is present. What is the maximum kinetic energy of the mass?

(c) If the mass is subject to a drag force $F_d = -\gamma v$, (with $\gamma^2 << 8am$) and is released from rest at x = 0 at t =

Problem 14:

A quantum mechanical system can be described by a Hamiltonian $\mathrm{H} = \begin{pmatrix} 0 & \mathrm{a} \\ \mathrm{a} & 0 \end{pmatrix}$, where a is a real number, in the orthonormal basis $|1\rangle$ and $|2\rangle$.

(a) What are the energy eigenvalues and normalized eigenvectors of this Hamiltonian?

(b) What is the probability for finding the system in state $|2\rangle$ at time t > 0 if the system is known to be in state $|1\rangle$ at t = 0?

Problem 15:

Find the heat capacity $\mathrm{C_v}$ of:

(a) one mole of argon in the gas phase,

(b) a carbon dioxide (CO_2) molecule in the gas phase (a linear molecule),

(c) an alcohol molecule (C_2H_6O) in the gas phase (NB, not a linear molecule),

(d) 0.1 mol of lead atoms in a lead crystal,

all in the high temperature limit. Assume ideal gases in (a) through (c).