August 2019 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $q_e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^8$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\,400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\,7685 * 10^{-3}$ m K
Plank radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\,\lambda)) - 1]^{-1}$

Useful integral: $\int \sin^2(x)\,dx = x/2 - ¼ \sin(2x)$

Section I:
Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

Problem 1:
A baseball of mass 200 g is pitched at a speed of 30 m/s towards the batter.
The batted ball's velocity is 60 m/s in the opposite direction.
The ball remained in contact with the bat for 2 ms. What is the average force applied to the bat?

Problem 2:
The noise level in an empty examination hall is 40 dB. When 100 students are writing an exam, the sounds of heavy breathing and pens traveling rapidly over paper cause the noise level to rise to 60 dB. Assuming that each student contributes an equal amount of noise power, find the noise level when 50 students have left.

Problem 3:
A beam of light with $\lambda = 600$ nm is shone through a double slit to a screen 5 m away. The first maximum is 2 cm from the central maximum.
(a) How far apart are the slits?
(b) What would happen if you shot photons through the slits one at a time?

Problem 4:
A particle of mass m moves in one dimension under the influence of a force
$F(x, t) = (k/x^2)\exp(-t/\tau)$, where k and τ are positive constants.
Compute the Lagrangian, and discuss whether energy is conserved in this system.

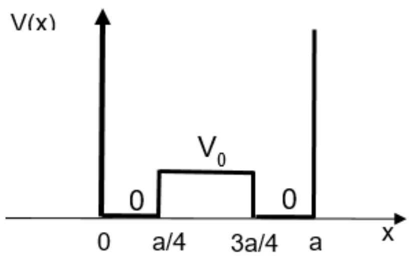
Problem 5:
A spacecraft is measured by an observer on earth to have a length of 53 m as it flies toward the earth with a speed $1.7*10^8$ m/s. The spacecraft then lands, and its length is again measured by the observer on the ground, this time while the spacecraft is at rest on the ground. What result does the observer now get for the length?

Problem 6:
An unidentified, hydrogenic ion (with all but one electron removed) is observed to emit photons at 2.70 nanometers, 2.85 nanometers and 3.38 nanometers. If this emission represents transitions to the ground state of the ion, of what element is the ion made.

Problem 7:
Imagine a sample of leather from a tomb is burned to obtain 0.12 g of carbon. The measured activity of the sample due to ${}^{14}_{6}C$ decay is 0.012 Bq (about 43 decays per hour).
How old is the sample?
For ${}^{14}_{6}C$, $t_{½}$ = 5730 y, and about 1 carbon atom in $7.7*10^{11}$ is ${}^{14}_{6}C$.

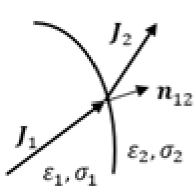
Problem 8:

Using perturbation theory to first order, find the corrected energy of the ground state (n = 1) of the "modified" infinite square well shown in the figure.



Problem 9:
The proper half life of a muon is $1.5*10^{-6}$ s. You have 10^6 muons moving at v = 0.8 c starting 720 above the Earth and moving directly towards the surface. How many reach the surface of the Earth?

Problem 10:
Two connected conductors have permittivities ε_1 and ε_2 and conductivities σ_1 and σ_2, respectively. A steady current through the interface has density $\mathbf{j}_1$ and $\mathbf{j}_2$. Calculate the free charge density α on the interface of the two conductors. (Assume the microscopic version of Ohm's law holds.)



Section II:

Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:

Consider three (non-interacting) particles in thermal equilibrium, in a one-dimensional harmonic oscillator, with a total energy $E = (7/2)\hbar\omega$. (Reminder: the energies of the one-particle states are $E_n = \frac{1}{2}\hbar\omega + n\hbar\omega$, with $n = 0, 1, 2, \ldots$).

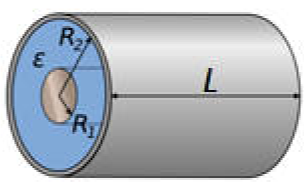
(a) If they are distinguishable particles (all with the same ω), what are the possible occupation-number configurations, and how many distinct (three particle) states are there for each one? If you picked a particle at random and measured the energy, what values might you get, and what is the probability of each one? Check that the sum of probabilities is 1.

(b) Repeat for identical fermions, ignoring spin.

(c) Repeat for identical bosons, also ignoring spin.

Problem 12:

A cylindrical capacitor of length L is formed of an inner conducting cylinder of radius R_1 surrounded concentrically by a thin conducting cylindrical shell of radius R_2. The medium dielectric in between the two cylinders has permittivity ε. Let $R_2 \ll L$.



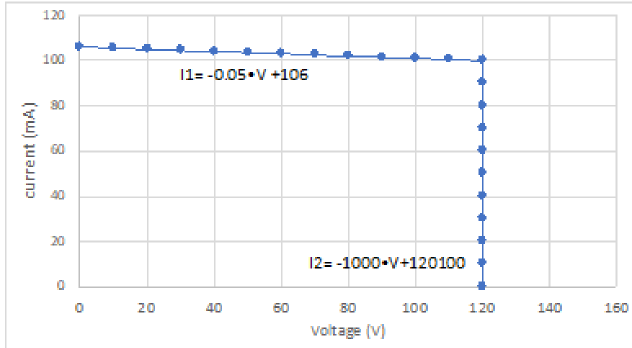
(a) Assume the capacitor has a charge Q. Calculate the electric field E between the two cylinders.

(b) Calculate the capacitance C.

(c) Now connect the capacitor to a battery with voltage V and then pull out the dielectric by a distance of x ($x < L$). What is the force that has to be applied on the dielectric? (assume there is no friction.)

Problem 13:

Consider the current versus voltage plot provided by a manufacturer of a power supply. Notice how it has two distinct regimes, defined as I_1 and I_2 (currents are in mA, 10^{-3} A).



$I_1 = (-0.05\ V + 106)$ mA,
$I_2 = (-1000\ V + 120100)$ mA.

(a) What are the Thevenin voltage, Norton current, and Thevenin resistance of both operational regimes?
(b) Calculate the maximum operational current, where the power supply's regimes 1 and 2 have equal currents (you can cross-check your result in the figure).
(c) Draw the Thevenin equivalent circuit of the second regime.
(d) What is the smallest resistor load (R_L) we can connect to the power supply before it changes to regime 1?

Problem 14:

A 3-kg sphere dropped through air has a terminal speed of 25 m/s. Assume that the drag force is proportional to the velocity. The sphere is attached to a spring of force constant 400 N/m and starts to oscillate with an initial amplitude of 20 cm.
(a) When will the amplitude be 10 cm?
(b) How much energy will have been lost when the amplitude is 10 cm?

Problem 15:

Derive Bohr's model of the atom.