Fall 2010 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 \times 10^{-34}$ Js, $\hbar = 1.05457266 \times 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 \times 10^{-23}$ J/K
Elementary charge: $e = 1.60216487 \times 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 \times 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 \times 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 \times 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 \times 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 \times 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 \times 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 \times 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 \times 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\,400 \times 10^{-8}$ W m^{-2} K^{-4}
Wien wavelength displacement law constant: $\sigma_w = 2.897\,7685 \times 10^{-3}$ mK

Units: 1 cal = 4.186 J

Section I:

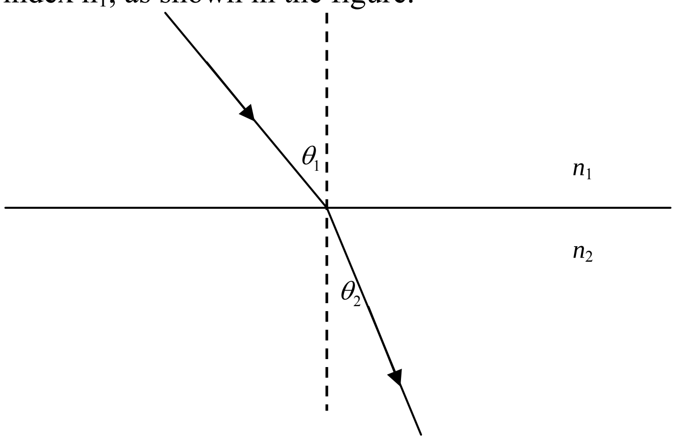
Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

Problem 1:

A 100 kg man climbs a vertical rope with an acceleration of 12 cm/sec^2. Find the tension in the rope.

Problem 2:

Propagation of Errors: Snell's law relates the angle of refraction θ_2 of a light ray travelling in a medium of index of refraction n_2 to the angle of incidence θ_1 of a ray travelling in a medium of index n_1, as shown in the figure.



From the knowledge that $n_1 = 1.000$ and from independent experimental measurements of $\theta_1 = (22.03 \pm 0.8)°$, and $\theta_2 = (10.45 \pm 0.8)°$, find

(a) n_2 ;

(b) the percentage errors in $\sin\theta_1$ and in $\sin\theta_2$;

(c) the uncertainty in n_2.

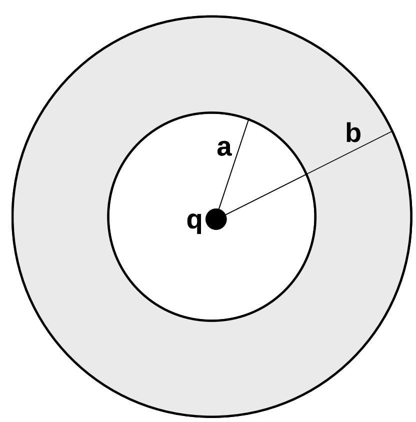
Problem 3:

A hollow uncharged spherical conducting shell has an inner radius *a* and an outer radius *b*. A positive point charge *q* is in the cavity at the center of the sphere.

(a) Find the charge on each surface of the conductor (surface *a* and surface *b*).

(b) Find the electric field everywhere.

(c) Find the potential everywhere, assuming that V = 0 at infinity.

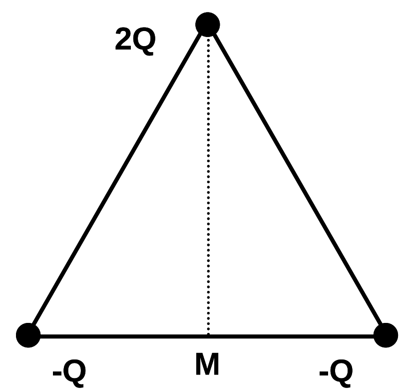


Problem 4:
A cup contains 0.2 kg of tea and is heated to 90 ^{o}C. The specific heat capacity of tea is ~ 1100 cal/(kg K). If the tea is allowed to cool to 20 ^{o}C calculate the change in entropy (J/K). Also calculate the entropy change in the room (T_{room} = 20 ^{o}C) and show that the total change in entropy is positive.

Problem 5:
(a) A monochromatic point source of light radiates isotropically (equally in all directions) with a power of 25 W at a wavelength of 5000 Å. A plate of metal is placed 100 cm from the source. Atoms in the metal have a radius of 1 Å. Assume that the atom can <u>continually</u> absorb light. The work function of the metal is 4 eV. How long is it before an electron is emitted from the metal?
(b) Is there sufficient energy in a single photon in the radiation field to eject an electron from the metal? <u>Explain.</u>

Problem 6:
Three charges, -Q, -Q and + 2Q, where Q = 1 microCoulomb, are arranged on an equilateral triangle with sides of length L = 1 m as shown. Point M is on the midpoint of the line joining the two negative charges. Potentials are referenced to zero at infinity.

(a) Compute the force (magnitude with correct units and direction) on the charge +2Q due to the other two charges.
(b) Compute the electric field (magnitude with correct units and direction) at point M produced by the three charges.
(c) Calculate the energy necessary for bringing a +Q charge from infinity to point M.
(d) Calculate the total energy necessary to assemble the whole system of four charges.



Problem 7:
An apparent limit on the lowest temperature achievable by laser cooling is reached when an atom's recoil energy upon absorbing or emitting a single photon is approximately equal to its total kinetic energy (3/2 kT). Calculate this "recoil temperature" for the Rubidium atom (m = 85 u, u = 1.66 x 10^{-27}kg) if the wavelength of the photon in resonance with Rubidium is 780 nm.

Problem 8:

At t = 0 a particle has a wave function $\psi(x) = A\exp(-x/a)$ for $x > 0$, $\psi(x) = 0$ for $x < 0$. Determine the normalization constant A and find the probability that the particle is found within a distance a from the origin at t = 0.

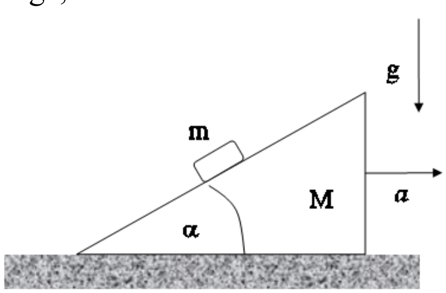
Problem 9:
A spaceship whose rest length is 350 m has a speed of 0.82c in a certain reference frame. A micrometeorite, also with a speed of 0.82c in this same frame, passes the spaceship on an anti-parallel track. According to an astronaut in the spaceship, how long does it take the micrometeorite to traverse the entire length of the spaceship?

Problem 10:
A transverse wave propagates in an infinitely long wire of mass per unit length 100 g/m and with a tension of 10 N. An observer next to the wire notices 10 crests passing him in a time of 2 seconds moving to the left.
(a) What is the wave velocity on the wire?
(b) What is the frequency of the wave?
(c) What is the wavelength?
(d) If at t = 0 and x = 0 the displacement assumes its maximum value of 1 mm, what is the equation of the wave?
(e) The wire is now fixed, under the same tension, at two points separated by 0.15 meters. What is the frequency of the third harmonic?

Section II:

Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:
A wedge of mass M rests on a horizontal, frictionless surface. A small mass m is placed on the wedge, whose surface is frictionless. Find the horizontal acceleration a of the wedge.



Problem 12:
A coaxial cable consists of an inner conductor of radius a and outer conductor of radius b. Current I flows along one conductor and back along the other.
(a) Calculate the inductance per unit length (L) of the cable.
(b) Calculate the total energy per unit length (U) of the cable.
(c) Verify that $U = \frac{1}{2}LI^2$.

Problem 13:

As shown in the figure, a uniform thin rod of weight W is supported horizontally by two supports, one at each end. At t = 0, one of these supports is removed. Find the force on the remaining support immediately thereafter.

Problem 14:
The distance between points A and B along a telegraph line, consisting of a pair of conducting wires, is L. There is a single leak between the two wires at a distance x from point A. If a voltage V_A' is applied between the two wires at point A, the voltage between the two wires at point B is V_B'. However if a voltage V_B'' is applied between the two wires at point B, the voltage at A is V_A''. Assuming the resistance per unit length of both wires is ρ derive a relationship giving the distance x of the leak as a function of L, V_A', V_B', V_A'' and V_B''. Check your answer by showing that x = 19 miles if L = 50 miles, V_A' = 200 volt, V_A'' = 40 volt, V_B' = 40 volt, V_B'' = 300 volt.

Problem 15:
Consider a two-state system goverened by the Hamiltonian H with energy eigenstates $|E_1\rangle$ and $|E_2\rangle$, where $H|E_1\rangle = E_1|E_1\rangle$ and $H|E_2\rangle = E_2|E_2\rangle$.
Consider also two other states,

$$|x\rangle = \frac{|E_1\rangle + |E_2\rangle}{\sqrt{2}} \quad \text{and} \quad |y\rangle = \frac{|E_1\rangle - |E_2\rangle}{\sqrt{2}}.$$

At time t = 0 the system is in state $|x\rangle$. At what subsequent times is the probability of finding the system in state $|y\rangle$ the largest, and what is that probability?