Fall 2012 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded.
Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma =$ 5.670 400 $* 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\ 7685 * 10^{-3}$ m K

Index of refraction of water: $n = 1.33$

Conversions:
1 kcal = 4186 J

Section I:
Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

Problem 1:

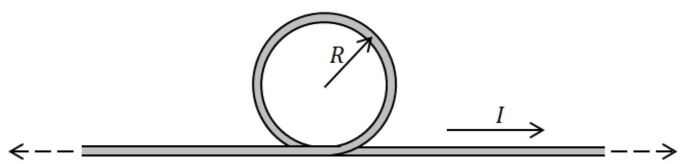
Find the minimum rotational frequency (i.e. the number of rotations per second, *not* the angular frequency) required to rotate a bucket containing 4 liters of water in a circular path in a vertical plane at an arm's length (about 70 cm) without spilling water.

Problem 2:

Three moles of hydrogen gas are confined to a volume of 0.4 m^3 at a temperature of 24°C.
(a) What is the average kinetic energy per molecule?
(b) What is the root mean square speed of the molecules?

Problem 3:

Find the magnetic field at the center of a circular loop of radius R that is formed in a long straight thin wire that carries current I; use the SI system of units.



Problem 4:

A steel disk A with radius R moves with speed v = 10 m/s when it collides with a second identical disk B at rest. The collision is elastic and has an impact parameter "b". After the collision, the speed of disk A is equal to 5 m/s. What is the value of the impact parameter "b"? Neglect friction.

Problem 5:

Find the magnetic force on charge q moving with velocity **v** due to another charge q' moving with velocity **v**' when the positions of the charges are **r** and **r**', respectively.

Problem 6:

(a) 100 kJ of electrical energy is dissipated in 1 liter of water. If the water is initially at 15 ^{o}C, and if that is also the lowest temperature available, what fraction of that energy is re-convertible into work?
(b) If the same amount of electrical energy were dissipated in a giant lake at 15 ^{o}C, what would be the change in entropy of the universe?

Problem 7:

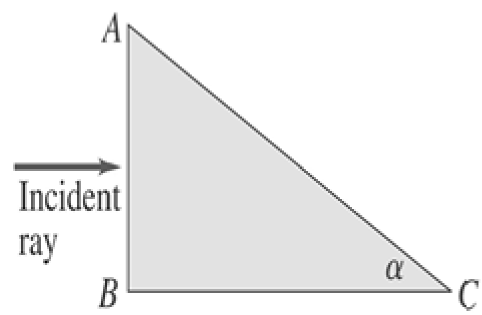
Assume that the nucleus of a hydrogen atom is a sphere of radius b. In the limit where $b << a_0$ (where a_0 is the Bohr radius), what is the probability that an electron in the ground state of hydrogen will be found inside the nucleus?

Problem 8:

Two identical particles are each moving with a speed 0.8 c and are on a collision course. The velocity vectors of the two particles are at 90 degrees to each other. After the collision the two particles create one new particle. What is a ratio of the rest mass of the new particle to the sum of the rest masses of the initial particles?

Problem 9:

Light is incident along the normal on face *AB* of a glass prism of index of refraction 1.52, as shown in the figure. Find the *largest* value the angle α can have such that there is no light refracted out of the prism at face *AC* if:
(a) the prism is immersed in air
(b) the prism is immersed in water



Problem 10:

Consider a simple clock consisting of two mirrors parallel to each other that reflect a light pulse back and forth without attenuation of the light, with the period of each "tick" equal to the time for light to travel between the mirrors. Now suppose the clock is placed in uniform motion in a direction parallel to the two mirrors, as observed by a stationary observer. Derive the time dilation formula of special relativity from considering what the stationary observer measures for the length of the clock ticks.

Section II:
Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:
The coefficient of viscosity μ_v is defined as the viscous force per unit area per unit velocity gradient

$$\mu_v = \text{force/area/velocity gradient}$$

Assume that μ_v is a function of mass, radius, and speed of the gas molecules of the form

$$\mu_v = \kappa\, m^x\, r^y\, v^z,$$

in which κ is a dimensionless constant.
Using dimensional analysis find the powers x, y, z and from this determine the dependence of μ_v on the temperature.

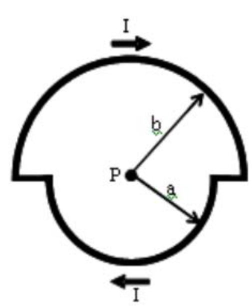
Problem 12:

The Compton effect, discovered by Arthur Compton in 1923, provided direct confirmation of the quantum nature of x-rays.
Consider the scattering of an x-ray photon with momentum $\vec{p}$ by an electron which, for simplicity, we assume to be at rest before the scattering. The magnitude of the photon momentum is related to its wave length λ according to $p = h/\lambda$, where h is Planck's constant. After the scattering, the photon has momentum $\vec{p}'$; the corresponding wave length is $\lambda' = h/p'$.

Making use of the equations of conservation of energy and momentum in the scattering event, obtain the Compton formula for the change in wave length $\lambda' - \lambda$. (Your result should depend on the angle between the two momenta $\vec{p}$ and $\vec{p}'$.)

Problem 13:

The conductor in the figure below is a wire that carries a current I in the direction indicated by the arrows. The conductor consists of two circular arcs and two straight sections. What is the magnitude of the net magnetic field at point P (the center of the circles).



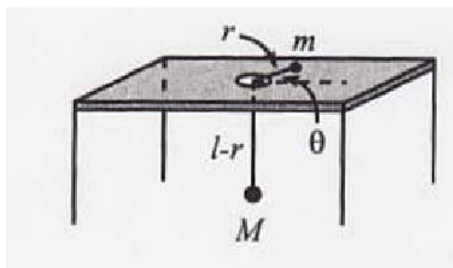
Problem 14:

Assume three particles and three distinct one particle states ($\psi_a(x)$, $\psi_b(x)$, $\psi_c(x)$). Describe in detail the possible three particle states that can be constructed if:

(a) The three particles are distinguishable,
(b) The three particles are identical fermions, and
(c) The three particles are identical bosons.

Problem 15:

A mass *m* is free to slide on a frictionless table and is connected, via a massless string of length *l* that passes through a hole in the table, to a mass *M* that hangs below (see Figure). Assume that *M* only moves vertically and that the string always remains taut.



(a) Find the equations of motion for the variables *r* and θ shown in this figure.
(b) Under what condition does *m* undergo a circular motion? Show that in that case, the gravitational force on *M* exactly accounts for the centripetal acceleration of *m*.