August 2014 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\,400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\,7685 * 10^{-3}$ m K
Plank radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\,\lambda)) - 1]^{-1}$

Useful integrals:

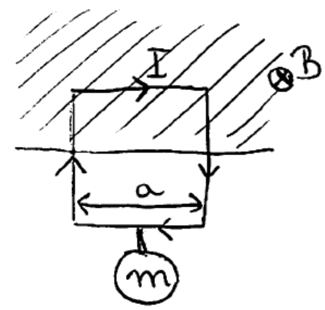
$$\int_0^\infty x^{2n} e^{-ax^2}\, dx = \frac{(2n)!}{n! 2^{2n+1}} \sqrt{\frac{\pi}{a^{2n+1}}} \qquad \int \frac{x\, dx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2}$$

$$\int_0^\infty x^{2n+1} e^{-ax^2}\, dx = \frac{n!}{2a^{n+1}}$$

Section I:
Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

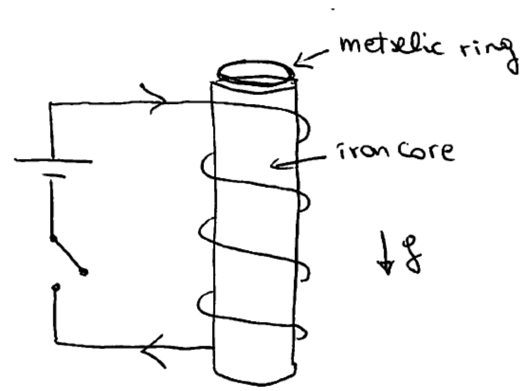
Problem 1:
5 moles of gas in a cylinder undergo an isobaric expansion starting at 293K. The cylinder is initially 50 cm tall. The radius of the cylinder is 10 cm and Δy is 1 cm. How much work is done by the gas?

Problem 2:
Consider the setup in the figure.
There is a weight of mass m hanging from a wire of negligible mass. A current circulates in the wire. The gravitational acceleration points down and has magnitude g. A uniform magnetic field **B** points into the page in the shaded region. For what value of I will the weight simply hang there, suspended in mid-air?



Problem 3:
The Balmer α line of hydrogen, with a wavelength of $\lambda = 656.3$ nanometers, results from a transition from the second excited state to the first excited state of hydrogen.
(a) What is the wavelength of the Lyman α line that results from the transition from the first excited state to the ground state?
(b) What would be the wavelength of the transition from the second excited state to the first excited state in 7 times ionized oxygen?

Problem 4:
If you wind a solenoid coil around an iron core as shown in the figure, place a metal ring on top and then close the circuit, the ring will jump several feet in the air. Why?
(Only a qualitative discussion is asked for.)



Problem 5:
Two police cars have identical sirens that produce a frequency of f = 570 Hz. A stationary listener is standing between two cars. One car is parked and the other is approaching the listener and both have their sirens on. The listener notices 2.6 beats per second. Find the speed of the approaching police car (the speed of sound is v = 344 m/s).

Problem 6:

Sirius A is the brightest star in the night sky, with the peak of its spectral emittance at a wavelength of 291 nanometers.

(a) If one makes the reasonable assumption that the star radiates as a blackbody, what is its effective temperature?

(b) If the flux measured on Earth from Sirius A is $1.17 * 10^{-7}$ W/m^2, and the distance is known to be 8.6 light-year, what is the radius of Sirius A?

Problem 7:

A hauling truck is traveling on a level road. The driver suddenly applies the brakes, causing the truck to decelerate by an amount g/2. This causes a box in the rear of the truck to slide forward. If the coefficient of sliding friction between the box and the truck bed is 1/3, find the acceleration of the box relative to

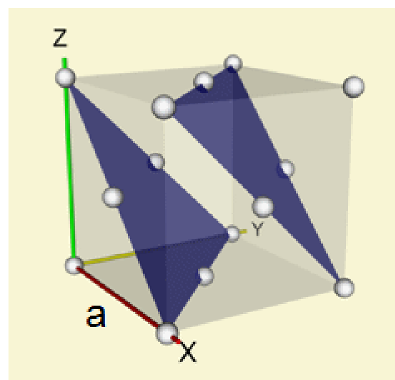
(a) the truck and

(b) the road.

Problem 8:

(a) An incident monochromatic x-ray beam with wavelength λ = 1.9 Å is reflected from the (111) plane in a 3D solid with a Bragg angle of 32^o for the n = 1 reflection. Compute the distance (in Å) between adjacent (111) planes**.**

(b) Assuming that the solid has a fcc lattice, use the result from part (a) to compute the lattice constant a (in **Å**).



(111) planes, FCC lattice

Problem 9:

The terminal speed of a freely falling object is v_t (assume the drag force is proportional to the speed of the object). When the object is suspended by a spring, while still influenced by the same drag force, the spring stretches by an amount x_0. Show that the frequency of oscillation of the object (when it is suspended by the spring) is ω, where $\omega^2 = g/x_0 - g^2/(4v_t^2)$

Problem 10:

N = 10^6 small, conductive, and widely separated spherical droplets are merged into one spherical drop. The radius of each droplet is r = $5.0*10^{-4}$ cm and the electrical charge of each droplet is q = $1.6*10^{-14}$ C.

(a) Find the potential of the large drop.

(b) How much work must be done by an external force to merge the droplets?

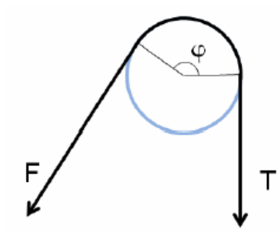
Section II:
Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:
For a quantum mechanical point particle in a 1-dimensional harmonic potential $U(x) = ½m\omega^2x^2$
(a) find the minimum (or "zero-point") energy using the lower limit of Heisenberg's uncertainty principle, $\Delta x\Delta p \geq \hbar/2$.
(b) For this zero-point energy of the particle, find the probability distribution by solving the time-independent Schroedinger equation for $\psi(x)$.

Problem 12:
A point mass m, at rest at the origin at t = 0 is acted on by a constant force $\mathbf{F} = ma\,\mathbf{i}$. Find the time T it takes for the mass to reach a speed v = 0.75c and the position of the mass at time T.

Problem 13:
An unknown particle X decays very quickly into a neutron (m = 940 MeV/c^2) with momentum p = (1000 MeV/c, 0, 0) and a positively charged pion (m = 140 MeV/c^2) with momentum p = (500 MeV/c, 500 MeV/c, 0). What is the mass of this unknown particle?

Problem 14:
A rope is wound around a cylinder of radius R with a contact angle φ (φ can be larger than 2π) to hold a massive object in place, for example to secure a ship for docking.
The object exerts a force F on one end of the rope and the other end of the rope held by a force T. The coefficient of static friction between the rope and the cylinder is k. Find the ratio F/T required to keep the object stationary as a function of φ.



Problem 15:
Consider a hollow sphere of radius R with a surface potential $\Phi(R,\theta) = k\sin^2\theta$, where k is a constant and θ is the usual polar angle relative to the z-axis.
Find the potential everywhere inside the sphere.
Hint:
$P_0(x) = 1$
$P_1(x) = x$
$P_2(x) = (3x^2 - 1)/2$
$P_3(x) = (5x^3 - 3x)/2$