January 2018 Qualifying Exam

Part II

Mathematical tables are allowed. Formula sheets are provided.

Calculators are allowed.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $q_e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\ 400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\ 7685 * 10^{-3}$ m K
Plank radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\ \lambda)) - 1]^{-1}$

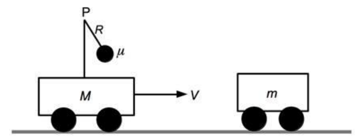
Hydrogen atom wave functions:
$R_{10}(r) = 2\ a_0^{-3/2} \exp(-r/a_0)$,
$R_{20}(r) = (2a_0)^{-3/2} (2 - r/a_0) \exp(-r/(2a_0))$,
$R_{21}(r) = 3^{-½}(2a_0)^{-3/2}(r/a_0) \exp(r/(2a_0))$,
$Y_{00} = (4\pi)^{-½}$, $Y_{1\pm1} = \mp(3/(8\pi))^{½}\sin\theta \exp(\pm i\varphi)$, $Y_{10} = (3/(4\pi))^{½}\cos\theta$.

Useful integrals:
$\int x\ dx/(ax^2 + bx + c)^{½} = (ax^2 + bx + c)^{½}/a - (b/(2a))\int dx/(ax^2 + bx + c)^{½}$
$\int dx/(ax^2 + bx + c)^{½} = a^{-½} \ln(2a^{½} (ax^2 + bx + c)^{½} + 2ax + b)$
$\int_0^\infty x^n e^{-ax} dx = n!/a^{n+1}$

Solve 6 out of the 8 problems! (All problems carry the same weight)

Problem 1:

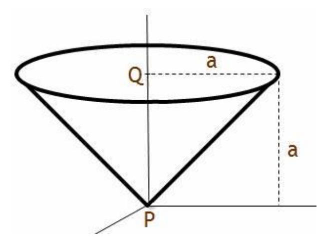
A cart of mass M has a pole mounted on it as illustrated in the figure. Assume the pole mass is negligible. A ball of mass μ hangs by a massless string, of length R, attached to the pole at point P.



(a) Suppose the cart (of mass M) and the ball are initially at rest, with the ball hanging in its equilibrium position. Calculate the minimum velocity that must be imparted to the ball for it to rotate in a circle of radius R in the vertical plane.
(b) Now suppose the cart and ball have initial velocity V towards the right. The cart crashes into a stationary cart of mass m and sticks to it. Find the velocity of the system after the collision. In this part and the next, neglect friction and assume that $M, m \gg \mu$.
(c) Find the smallest value of the initial cart speed for which the ball can go in circles in the vertical plane following a collision.

Problem 2:

A conical surface (an empty ice-cream cone) carries a uniform surface charge σ. The height of the cone is a, as is the radius of the top.



(a) Find the electrical potential at point P (vertex of the cone).
(b) Find the electrical potential at point Q (center of the top of the cone).
(c) Find the potential difference between points P (the vertex) and Q (the center of the top).

Problem 3:

The usual wave functions and energies for the hydrogen atom assume that the nuclear charge is point-like. In fact, the proton has a finite size.
Assume that the proton is a sphere of uniform charge density with radius $r_p \ll a_0$, where a_0 is the Bohr radius.
(a) What is the probability that the electron will be found within the nucleus?
(b) Using first order perturbation theory, determine the change in the hydrogen binding energy. Please clearly identify the sign of this change.

Problem 4:
A river of width D flows on the northern hemisphere at a geographical latitude φ toward the north with a certain flow speed v_0. By which amount is the right bank higher that the left one?
First apply the equation of motion in a non-inertial frame to the problem at hand, and then use D = 2 km, v_0 = 5 km/h, and $\varphi = 45^o$ to find the super-elevation of the river for the given parameters.

$M_{Earth} = 5.97*10^{24}$ kg, R_{Earth} = 6378 km.

Problem 5:
A hydrogen atom in its ground state [(n, l ,m) = (1, 0, 0)] is placed between the plates of a capacitor. A time dependent but spatially uniform electric field (not potential!) is applied as follows:
E = 0 for t < 0,
$\mathbf{E} = \mathbf{E}_0 \exp(-t/\tau)$ for t > 0. $\mathbf{E}_0 = E_0\, \mathbf{k}$.
Using first-order time-dependent perturbation theory compute the probability for the atom to be found at $t >> \tau$ in each of the three p-states n= 2, l = 1, m = ±1 or 0).

Problem 6:
A spaceship is initially at rest with respect to frame S. At a given instant, it starts to accelerate with constant proper acceleration **a**. (The proper acceleration is the acceleration with respect to the instantaneous inertial frame the spaceship was just in. Equivalently, if an astronaut has mass m and is standing on a scale, then the scale reads a force of **F** = -m**a**.) What is the relative speed of the spaceship and frame S when the spaceship's clock reads time t?

Problem 7:

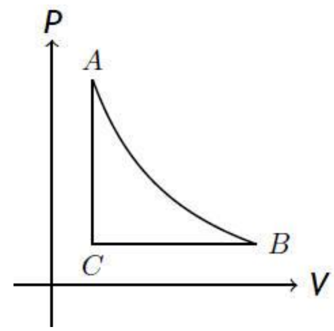
A ball with radius a and permeability μ is placed in a previously uniform magnetic field $\mathbf{B}_0$.

(a) Find the fields **B** and **H** and the induced magnetic dipole moment of the ball.

(b) If the ball has permanent magnetization $\mathbf{M}_0$, calculate the field **B** and **H** of the ball.

Problem 8:

One mole of a monatomic ideal gas is driven around the cycle A B C A shown on the PV diagram below. Step AB is isothermic, with a temperature T_A = 500 K. Step BC is isobaric, and step CA is isochoric. The volume of the gas at point A is V_A = 1 liter, and at point B is V_B = 4 liter.



(a) What is the pressure P_B at point B?

(b) What is the net work done by the gas in completing one cycle A B C A?

(c) What is the entropy change S_C - S_B?

Provide numerical answers in SI units.