January 2019 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $q_e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\ 400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\ 7685 * 10^{-3}$ m K
Plank radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\ \lambda)) - 1]^{-1}$

Section I:
Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

Problem 1:

What is the maximum speed of a point mass sliding from rest without friction from the top of the cycloid described by the equations: $x = R(\gamma + \sin\gamma)$ and $y = R(1 + \cos\gamma)$ where $-\pi < \gamma < \pi$. The gravitational acceleration **g** points in the negative y-direction.

Problem 2:

The operators A and B are represented by matrices in some basis.

$$A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}, \quad B = \begin{bmatrix} c & b \\ b & c \end{bmatrix}.$$

(a) Do A and B commute?
(b) Find the normalized eigenvectors of A and B.

Problem 3:

Consider an ideal gas of N particles at temperature T, confined to a two-dimensional surface. Suppose, in addition, that each particle has one internal rotational degree of freedom. What is the internal energy U of the gas according to the equipartition theorem?

Problem 4:

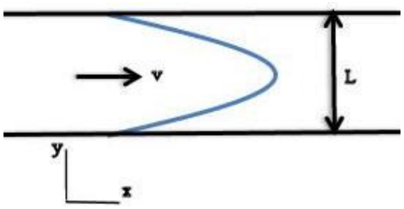
A rope of total length 5 m is coiled on a table with 0.8 m of the rope hanging vertically down off the edge. The total mass of the rope is 0.5 kg. How much work is required to place all the rope on the table?

Problem 5:

The emission spectrum of hydrogenic Lithium ions is measured and the wavelength of a series of emission lines are recorded. The longest line in that series has a wavelength of 450.25nm. What is the wavelength of the shortest line in that series?

Problem 6:

A boat is moving across a river of constant with L. The boat moves with constant velocity $\mathbf{v}_1$ relative to the water and perpendicular to the current. The current is parallel to the the river bank and the speed v of the water of depends on the distance from the river bank as $v = v_0 \sin(\pi y/L)$, where v_0 is a constant. Find the velocity vector $\mathbf{v}(x,y)$ of the boat with respect to the river bank and the trajectory y(x) of the boat. Use the orientation of the coordinate axes shown below.

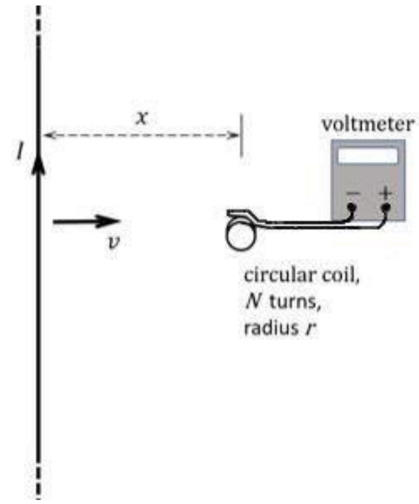


Problem 7:

Consider a muon particle, which is a heavy, unstable relative of the electron with a rest mass of $m_\mu \approx 0.1\ \mathrm{GeV}/c^2$. A muon decays typically to an electron and two neutrinos and has an average lifetime $\tau \approx 2.2\ \mu s$ in its rest frame. Suppose that we have a beam of fast-moving muons (generated in a particle accelerator, for example). We use this beam in the lab and find that the lifetime as measured in the lab is $\tau' \approx 10\ \mu s$.

(a) What is the speed of the muons in the beam, as measured in the lab?
(b) What is the energy of the muons in their rest frame?
(c) What's the kinetic energy and momentum of the muons as measured in the lab?
(d) How far do the muons travel on average in the lab before decaying?

Problem 8:



A long straight wire (extending indefinitely in the ±y direction) is carrying current I = 5 A in the +y-direction and moving with speed v = 2 m/s in the +x-direction toward a small circular conducting coil having N = 12 closely packed turns of radius r = 2 cm. The ends of the coil are connected to a voltmeter, and the coil lies in the xy plane, centered at the origin.

(a) At the instant that the wire is distance x = 0.2 m from the center of the coil, what is the magnitude of the reading of the voltmeter? Express your result both symbolically and numerically. The impedance of the voltmeter is much, much higher than that of the coil and connecting wires. Assume x is large enough that the field is approximately uniform over the area of the coil and ignore the emf in the loop formed by the connection to the meter.

(b) Assuming a small current (due to the high but non-infinite impedance of the voltmeter) flows in the coil, will that current be clockwise or counter-clockwise as viewed from a point on the +z axis?

Problem 9:

An elevator car of maximum mass mass m_0 when fully loaded is connected to a counter balance of the same mass by a cable of length L.

(a) Derive an expression for the cross sectional area A of the cable such that the yield strength σ_y, (a stress, force/area) of the cable is not exceeded.

(b) Let ρ be the mass density (mass per unit volume) of the cable and the acceleration due to gravity (9.8 m/s^2). Assume that m_0 = 8800 kg and the elevator car carries its load up a skyscraper of height 5 km (!). Calculate the minimum cross sectional area of the cable without exceeding σ_y for the case of a steel cable.

Use σ_{ysteel} = 1380 MPa and ρ_{steel} = 7.7 g/cm^3.

Problem 10:

A rocket starts from rest in free space by emitting mass with a constant speed with respect to the body of the rocket. At what fraction of its initial total mass has the rocket's momentum reached a maximum?

Section II:

Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:

The free energy, F, of a magneto-elastic material is the sum of magneto-elastic and strain energy terms.

(a) Assuming a simple form for the magneto-elastic energy term of $\gamma\varepsilon M^2$, where γ is the magneto-elastic coupling coefficient, ε is the strain and M is the magnetization, write an expression for F in terms of γ, ε, M^2 and Young's modulus, Y.

(b) Assuming $\gamma > 0$, sketch the two contributions to F as a function of ε from negative (compression) to positive (tension), and their sum (= F).

(c) Is the equilibrium value of strain compressive or tensile?

(d) Assuming equilibrium conditions with respect to ε, i.e. $dF/d\varepsilon = 0$, derive an expression for ε in terms of M^2, Y, and γ.

(e) To lowest order in M, does the equilibrium value of strain depend on the sign of M?

Problem 12:

For neutron with energy 0.025 eV, the values of the total scattering cross-section σ_{sc} and the absorption cross-section σ_{abs} for atoms of nitrogen and oxygen are:

σ (barns)	σ_{sc}	σ_{abs}
Nitrogen	11.5	1.8
Oxygen	4.2	~0

Estimate the attenuation of a beam of neutrons of 0.025 eV in 1 m of dry air at a temperature of 20^o C and and atmospheric pressure. The concentration ratio of oxygen/nitrogen is 20%/80%.

Problem 13:

Positive charge is distributed uniformly within the volume of a sphere having radius R and charge density ρ_0.

(a) Calculate the electric field and the potential outside the sphere.

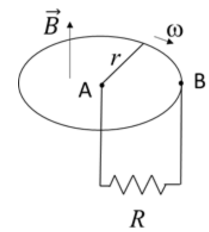
(b) Calculate the electric field and the potential within the sphere.

(c) An electron with charge q_e and mass m_e oscillates about the center of this sphere (within R). Find the period of the oscillations. Assume electron has negligible effect on the charge distribution ρ_0.

Problem 14:

Find the relative decrease in the photon frequency $\Delta\omega/\omega_0$ in the gravitational field of the Earth (gravitational redshift) when a photon moves up from the Earth's surface a distance h = 20 m.

Problem 15:



One end of a conducting rod rotates with angular velocity ω in a circle of radius r making contact with a horizontal, conducting ring of the same radius. The other end of the rod is fixed. Stationary conducting wires connect the fixed end of the rod (A) and a fixed point on the ring (B) to either end of a resistance R. A uniform vertical magnetic field **B** passes through the ring.

(a) Find the emf produced in the system and its time dependence. [Hint: find Lorentz force acting on the charges in the moving rod].
(b) Determine direction of current flows in the circuit.
(c) Determine work needed to rotate the rod for one full turn.