January 2019 Qualifying Exam

Part II

Mathematical tables are allowed. Formula sheets are provided.

Calculators are allowed.

Please clearly mark the problems you have solved and want to be graded. Do only mark the required number of problems.

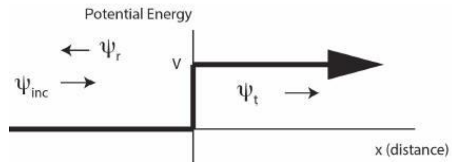
Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $q_e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\ 400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\ 7685 * 10^{-3}$ m K
Plank radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\ \lambda)) - 1]^{-1}$

Solve 6 out of the 8 problems! (All problems carry the same weight)

Problem 1:

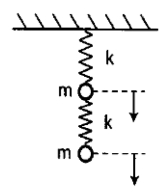
Particles with mass m and energy E in a beam can be represented by a plane wave function. Such particles are incident on a barrier at $x = 0$ with potential energy V ($V = 0$ for $x < 0$), as shown in the figure.



(a) Let k_0, k_1 be the wave vectors of the particles for $x < 0$ (left side of barrier) and $x > 0$, respectively. Write an expression for the incident wave function of the particles on the left side of the barrier.
(b) Write an expression for the reflected wave function.
(c) Using (a) and (b), write an expression for the total wave function on the left side.
(d) What is the magnitude of the wave vector of the particles in terms of E on the left side?
(e) Write an expression for the wave function of the particles on the right side of the barrier?
(f) What is the magnitude of the wave vector of the particles in terms of E and V on the right side?
(g) Derive an expression for the reflection probability R, in terms of E (not equal to 0) and V.
(h) What are the limiting values of R for $E < V$, $E >> V$, $E = V$ and $V = 0$?

Problem 2:

A particle of mass m is attached to a rigid support by a spring with a force constant k. At equilibrium, the spring hangs vertically downward. To this mass-spring combination is attached an identical oscillator, the spring of the latter being connected to the mass of the former.



(a) Show that by appropriate choice of coordinates and their zero-points, that the equations of motion can be expressed as
$md^2x_1/dt^2 + 2kx_1 - kx_2 = 0,$
$md^2x_2/dt^2 + kx_2 - kx_1 = 0.$
(b) Calculate the characteristic frequencies for one-dimensional vertical oscillations.
(c) Qualitatively describe the normal modes of the system with a short discussion plus drawings.
(d) Quantitatively compare the characteristic frequencies with the frequencies when one or the other of the particles is held fixed while the other oscillates. (Note: From part (a) we can see that there is a different frequency depending upon which one is held fixed.)

Problem 3:

A rectangular slab of unknown material is connected to a power supply, with polarities as shown in the figure. A current meter on the power supply indicates that the current running through the slab is constant and has a value I = 0.300 A. There is a constant and uniform magnetic field of B = 0.80 T perpendicular to the horizontal face of the slab (upward, in the +y direction, as shown). Two voltmeters are connected to the slab and read steady voltages, V_l and V_t, as shown. The connections for the transverse voltmeter, reading V_t = -0.00027 V, are carefully placed directly across from each other, and the connections for the longitudinal voltmeter, reading V_l = +4.80 V, are placed on the same side of the slab, a distance d = 10.0 cm apart. (Remember that a voltmeter reads a positive number if its 'positive' lead is connected to the higher potential location.) The slab is l = 15.0 cm long (in the z, or l, or longitudinal direction), w = 8.00 cm wide (in the x, or t, or transverse direction) and h = 1.00 cm thick (in the y direction). Assume that there is only one kind of mobile charges in this (electrically neutral) material, but it is not initially known whether they are positive (holes) or negative (electrons).

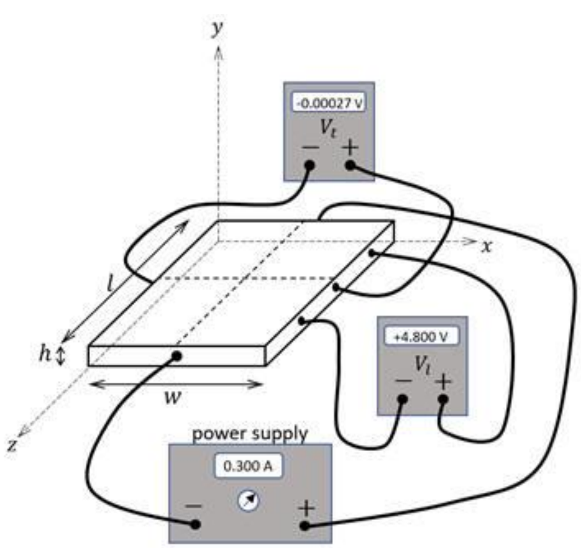
(a) From the information given, determine the sign of the mobile charges and the direction they move (±x, ±y, or ±z) in the steady state. Explain how you know.

(b) From the quantities given, develop a symbolic expression for the average drift speed of the mobile charges, and evaluate the drift speed numerically.

(c) Knowing the drift speed, determine the mobility, u, of the mobile charges. The mobility is defined as the ratio between the drift speed and the internal electric field that drives the current. Note that there are two contributions to the net electric field inside the slab. Think about which one drives the current. Express your result symbolically in terms of quantities given in the problem statement, and then evaluate it numerically.

(d) Assuming each mobile charge is singly charged (i.e., $|q| = q_e$), determine the mobile charge density, n, in terms of given quantities, both symbolically and numerically.

(e) What is the resistance (measured between the longitudinal contacts) of this slab, in ohms?



Problem 4:

A neutral pi meson with energy twice its rest mass ($E = 2mc^2$) is moving along the x-axis with respect to observer O. It decays into two photons. In the meson's rest frame the photons are emitted in opposite directions along the y' axis. Find the energy of the photons in both the pi-meson's rest frame and in the observer's frame, and the emission angles of the photons in the observer's frame.

Problem 5:

A particle of mass m is trapped in a 3-dimensional, infinite square well.
$V(x,y,z) = 0$, if x, y, and z are less than a and not negative,
$V(x,y,z) = \infty$, otherwise.
(a) What are the two lowest energy eigenvalues? Are these energies degenerate?
(b) The box is placed into a uniform gravitational field with gravitational acceleration **g** in the negative z-direction. Use perturbation theory to calculate first-order corrections to the energy eigenvalues from part (a).

Problem 6:

A charge q is located in the empty space above an infinite, flat, grounded conducting plate whose surface coincides with the plane at $z = 0$.
The coordinates of the charge are $(x, y, z) = (0, 0, d)$ with $d > 0$.
(a) Calculate the force acting on the charge.
(b) Calculate the electric field in the half-space $z > 0$.
(c) What is the surface charge density induced on the surface of the plate?
(d) How much work would be needed to pull the charge q away to $z = +\infty$ along the z-axis?

Problem 7:

Suppose we have a particle that can be in one of two energy levels $E = 0$ or $E = \varepsilon$. Assume the particle is in thermal equilibrium with a large thermal reservoir at temperature T, so that it occupies each of the two states according to the Boltzmann distribution
$P(E) \propto \exp(-E/(kT))$. Calculate the average energy $\langle E \rangle$ and the root-mean-square deviation from the average energy $\Delta E = \langle (E - \langle E \rangle)^2 \rangle^{1/2}$.

Problem 8:

Assume you have a sufficiently large concave spherical mirror with focal length F and outer diameter F. Assume the reflectance of the coated aluminum mirror is 88%. Is it possible to melt a small iron sphere of radius R by focusing the sun's rays with this mirror? Let R be the radius of the image of the sun, formed by the mirror. The melting point of iron is 1812 K, and the surface temperature of the sun is assumed to be $T_S = 5830$ K.