January 2013 Qualifying Exam

Part I

Calculators are allowed. No reference material may be used.

Please clearly mark the problems you have solved and want to be graded.
Do only mark the required number of problems.

Physical Constants:

Planck constant: $h = 6.62606896 * 10^{-34}$ Js, $\hbar = 1.054571628 * 10^{-34}$ Js
Boltzmann constant: $k_B = 1.3806504 * 10^{-23}$ J/K
Elementary charge: $e = 1.602176487 * 10^{-19}$ C
Avogadro number: $N_A = 6.02214179 * 10^{23}$ particles/mol
Speed of light: $c = 2.99792458 * 10^{8}$ m/s
Electron rest mass: $m_e = 9.10938215 * 10^{-31}$ kg
Proton rest mass: $m_p = 1.672621637 * 10^{-27}$ kg
Neutron rest mass: $m_n = 1.674927211 * 10^{-27}$ kg
Bohr radius: $a_0 = 5.2917720859 * 10^{-11}$ m
Compton wavelength of the electron: $\lambda_c = h/(m_e c) = 2.42631 * 10^{-12}$ m
Permeability of free space: $\mu_0 = 4\pi\ 10^{-7}$ N/A^2
Permittivity of free space: $\varepsilon_0 = 1/\mu_0 c^2$
Gravitational constant: $G = 6.67428 * 10^{-11}$ m^3/(kg s^2)
Stefan-Boltzmann constant: $\sigma = 5.670\,400 * 10^{-8}$ W m^{-2} K^{-4}
Wien displacement law constant: $\sigma_w = 2.897\,7685 * 10^{-3}$ m K

Plank radiation law: $I(\lambda,T) = (2hc^2/\lambda^5)[\exp(hc/(kT\,\lambda)) - 1]^{-1}$

Section I:
Work 8 out of 10 problems, problem 1 – problem 10! (8 points each)

Problem 1:

A 1000 Watt light bulb is left on inside a 1000 Watt refrigerator. Can the refrigerator cool below room temperature? Explain!

Problem 2:

A particle oscillates with simple harmonic motion along the x axis. Its displacement x from the origin varies with time t according to the equation
$x = (2\text{ m})*\cos((0.5\ \pi/\text{s})t + \pi/3)$,
where t is in seconds.
(a) Find formulas for the velocity and acceleration of the particle at any time.
(b) Find the maximum speed and the maximum acceleration of the particle.
(c) Find the displacement of the particle between the times t = 0 and t = 2 s.

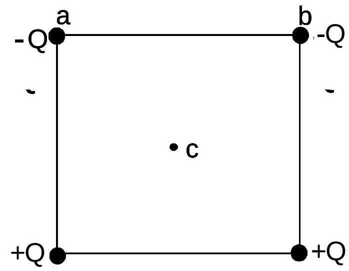
Problem 3:

A beam of monochromatic light traveling through air strikes the top of a flat slab of glass at an angle 60° to the normal, passes through the glass, and emerges from the bottom of the slab. The glass has a thickness t and a refractive index n = 1.52.
(a) What is the angle of refraction where the light enters the glass?
(b) Show that the beam emerging from the glass is parallel to the incident beam.

Problem 4:

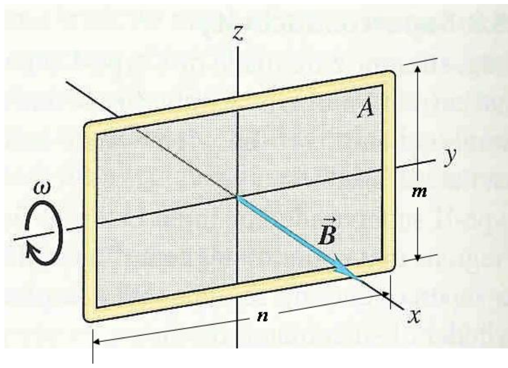
A child of mass m = 30 kg begins from rest and slides down a curved playground slide of height h = 3 m, reaching the bottom of the slide with a speed of v_f.
(a) If the slide is frictionless, find the speed v_f.
(b) If the slide has friction and v_f = 5 m/s, find the work done by friction.

Problem 5:



Two positive and two negative charges are arranged on the corners of a square whose sides have length L. Each charge has magnitude Q. The signs of the charges are as indicated. Point c is at the center of the square.
(a) What is the electrostatic potential at point c?
(b) What is the potential energy associated with the charge at point b due to the other three charges?
(c) How much work must be done to assemble the charges from ∞? Clearly explain your reasoning, indicate whether the result is + or -, and explain why.

Problem 6:



The loop in the figure has dimension m × n as shown. In time t is rotated by 180^o about the y-axis in a uniform magnetic field of magnitude B oriented along the x-axis as show.

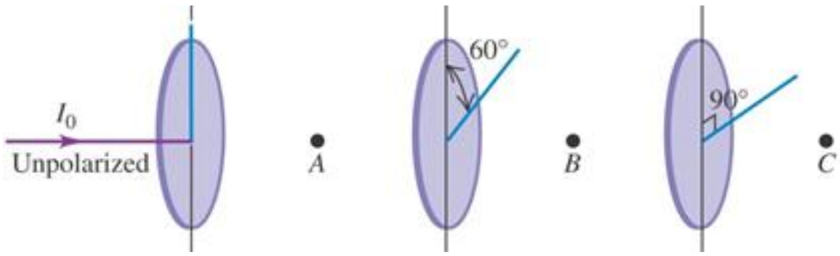
(a) What is the average emf induced in the loop during the rotation?
(b) If the rotation had been about the z-axis, what would the average emf have been?

Problem 7:

At some time t the wave function of a particle is a triangular hat wave function given by
$\Psi(x,t) = Ax/a$ for $0 < x < a$,
$\Psi(x,t) = A(b - x)/(b - a)$ for $a < x < b$,
$\Psi(x,t) = 0$ otherwise,
where A, a, and b are constants.
(a) Sketch Ψ and find the most probable location of the particle at time t.
(b) Determine the normalization constant A in terms of a and b.
(c) At time t, what are the probabilities of the particle being found left and right of a, respectively?
(d) What is $\langle x(t) \rangle$?

Problem 8:

A beam of unpolarized light of intensity I_0 passes through a series of ideal polarizing filters with their transmission axis turned to various angles, as shown in the figure.
(a) What is the light intensity (in terms of I_0) at points A, B, and C?
(b) If we remove the middle filter, what will be the light intensity at point C?



Problem 9:

Freezing or melting a gram of water/ice requires an energy transfer of about 334 Joules at a fixed temperature of 0 °C. A 1 g sample of water at 0 °C is frozen by placing it in contact with a thermal reservoir at −10 °C. The ice and unfrozen water are kept in contact and in equilibrium with each other until the water is completely frozen, at which time the ice is removed (before its temperature can drop below 0 °C).

(a) Find the change in entropy of the water/ice, the entropy change of the reservoir, and the total entropy change (of the "universe" of water plus reservoir).
(b) Suppose the process is reversed. 1 g of ice at 0 ^{o}C is melted by placing it in contact with a second reservoir. The water is removed as soon as it forms. What is the minimum temperature of the second reservoir?
(c) Choose (and state) a reasonable second reservoir temperature; then find the change in entropy of the water/ice, the entropy change of the reservoir, and the total entropy change (of the "universe" of water plus reservoir).
(d) After the freezing/melting cycle, you have a 1 g sample of water and the two reservoirs, just as you did at the start of this cycle of processes. What has changed?

Problem 10:

Consider a simple pendulum which on the earth has a period of 1.60 s. What would the period of this pendulum be on the surface of Mars, where the acceleration of gravity is $g_M = 3.71$ m/s^2?

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Section II:
Work 3 out of the 5 problems, problem 11 – problem 15! (12 points each)

Problem 11:

Find the gravitational field **Γ**(r) both outside an inside of an infinitely long cylinder of radius R with a uniform mass density ρ inside. (Note: **Γ** = **F**/m)

Problem 12:

Joe is using a calibrated spectrometer with a diffraction grating with a line density of 1200 lines per mm to measure the temperature of a melted metal. Two different wavelengths have their first-order maxima at 60 degrees and 45 degrees. The relative intensities of the two wavelengths are 10:1. What is the temperature of the metal?

Problem 13:

The Heisenberg Hamiltonian representing the exchange interaction between two spins ($\mathbf{S}_1$ and $\mathbf{S}_2$) is given by

$$\hat{H} = -2\xi(R)\hat{S}_1 \cdot \hat{S}_2,$$

where $\xi(R)$ is the exchange coupling constant and R the spatial separation between the two spins. Find the eigenstates and eigenvalues of the Heisenberg Hamiltonian.

Problem 14:

Two solid objects have the same temperature T initially and are found to have identical masses (of order grams). Both are made from material with specific heat capacity c_p. If one body is heated by a small amount to a temperature T + ΔT and then an identical force that is small in magnitude is applied to each body, which body accelerates at a higher rate? Estimate a_1/a_2.

Problem 15:
Find the fraction of the kinetic energy that is translational and rotational when
(a) a hoop
(b) a disc and
(c) a sphere rolls down an inclined plane of height h. Find the velocity at the bottom in each case. Compare with a block sliding without friction down the plane.