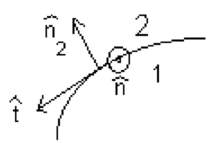
Formulas:

Mechanics

Motion in a non-inertial frame:	md**v**/dt = -∂U/∂**r** - md**V**/dt + m**r** $\times$ d**Ω**/dt - 2m**Ω** $\times$ **v** - m**Ω** $\times$ (**Ω** $\times$ **r**)
Lagrange's equations:	$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0$, $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j$, $Q_j = -\frac{\partial U}{\partial q_j}$ or $Q_j = -\frac{\partial U}{\partial q_j} + \frac{d}{dt}\left(\frac{\partial U}{\partial \dot{q}_j}\right)$
Hamilton's equations:	$\dot{q}_i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H}{\partial q_i}$, $H(q,p,t) = \sum_i p_i \dot{q}_i - L$, $\frac{\partial L}{\partial \dot{q}_j} = p_j$
Lagrange multipliers:	$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = \sum_l \lambda_l a_{lk}$, $\sum_k a_{lk} dq_k + a_{lt} dt = 0, \quad l = 1, \cdots, m$
Small oscillations:	$L = \frac{1}{2}\sum_{ij}\left(T_{ij}\dot{q}_i\dot{q}_j - k_{ij}q_iq_j\right)$ with $T_{ij} = T_{ji}, \quad k_{ij} = k_{ji}$ $\sum_j\left(k_{ij} - \omega_\alpha^2 T_{ij}\right) A_{j\alpha} = 0$ $q_j = \mathrm{Re}\sum_\alpha\left(C_\alpha A_{j\alpha} e^{i\omega_\alpha t}\right)$
Motion in a central potential:	$m\ddot{r} - \frac{M^2}{mr^3} = f(r), \quad$ or $\quad m\ddot{r} = -\frac{\partial U_{eff}(r)}{\partial r}$, with $f(r) = -\frac{\partial U(r)}{\partial r}$ $\frac{M^2u^2}{m}\left(\frac{d^2u}{d\phi^2} + u\right) = -f(u)$ $\dot{r} = \pm\sqrt{\frac{2}{m}\left(E - U_{eff}(r)\right)}$, $\phi = \phi_0 - \int \frac{M du}{\sqrt{2m\left(E - U_{eff}(u)\right)}}$ Kepler orbit: $\frac{p}{r} = 1 + e\cos\left(\phi - \phi_0\right)$
Two interacting particles:	CM frame: $L = \frac{1}{2}\mu\dot{\boldsymbol{r}}^2 - U(r), \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}, \qquad \boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2$

Relativistic kinematics:	$\begin{pmatrix} x_0' \\ x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0' \\ x_1' \\ x_2' \\ x_3' \end{pmatrix}$
4-vectors:	$(u_0, \boldsymbol{u}) = \left(\gamma c, \gamma\frac{d\boldsymbol{r}}{dt}\right) = \left(\frac{dx_0}{d\tau}, \frac{d\boldsymbol{r}}{d\tau}\right), \quad (u_0, \boldsymbol{u}) \cdot (u_0, \boldsymbol{u}) = c^2$ $\left(p_0, \boldsymbol{p}\right) = (mu_0, m\boldsymbol{u}) = \left(\gamma mc, \gamma m\frac{d\boldsymbol{r}}{dt}\right), \quad p_0 = \frac{E}{c} = \gamma mc, \quad \boldsymbol{p} = \gamma m\frac{d\boldsymbol{r}}{dt}$ $\left(p_0, \boldsymbol{p}\right) \cdot \left(p_0, \boldsymbol{p}\right) = m^2c^2 \quad \Rightarrow \quad E^2 = m^2c^4 + p^2c^2$
Transformation of velocities:	$u'_{\parallel} = (u_{\parallel} - v)/(1 - \mathbf{v}\cdot\mathbf{u}/c^2)$, $u'_{\perp} = u_{\perp}/(\gamma(1 - \mathbf{v}\cdot\mathbf{u}/c^2))$
Doppler shift:	$\omega' = \gamma\omega(1 - (v/c)\cos\theta)$
Relativistic collisions:	For each component p_μ of the 4-vector (p_0, p_1, p_2, p_3) we have $\sum_{particles\_in} p_\mu = \sum_{particles\_out} p_\mu, \quad \text{or} \quad \sum_i \left(p_i\right)_\mu = \sum_j \left(p_j\right)_\mu.$ For transformations between reference frames we have $(P_0, \boldsymbol{P}) \cdot (P_0, \boldsymbol{P}) = \left(P_0', \boldsymbol{P}'\right) \cdot \left(P_0', \boldsymbol{P}'\right),$ where $P_0 = \sum_{particles} p_0$ and $\boldsymbol{P} = \sum_{particles} \boldsymbol{p}$.

E&M

Maxwell's equations:	$\vec{\nabla}\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}$, $\vec{\nabla}\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$ $\vec{\nabla}\cdot\mathbf{B} = 0$, $\vec{\nabla}\times\mathbf{B} = \mu_0\mathbf{j} + \frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}$ $\vec{\nabla}\cdot\mathbf{D} = \rho_f$, $\vec{\nabla}\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$ $\vec{\nabla}\cdot\mathbf{B} = 0$, $\vec{\nabla}\times\mathbf{H} = \mathbf{j}_f + \frac{\partial\mathbf{D}}{\partial t}$ lih materials: $\vec{\nabla}\cdot\mathbf{E} = \frac{\rho_f}{\varepsilon}$, $\vec{\nabla}\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$, $\vec{\nabla}\cdot\mathbf{B} = 0$, $\vec{\nabla}\times\mathbf{B} = \mu\mathbf{j}_f + \mu\varepsilon\frac{\partial\mathbf{E}}{\partial t}$
Dipole field and potential:	$\mathbf{E}(\mathbf{r}) = \left[\frac{1}{4\pi\varepsilon_0}\right]_{SI}\frac{1}{r^3}\left[3\left(\mathbf{p}\cdot\hat{\mathbf{r}}\right)\hat{\mathbf{r}} - \mathbf{p}\right]$ $\phi(\mathbf{r}) = \left[\frac{1}{4\pi\varepsilon_0}\right]_{SI}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2}$ $\mathbf{F} = \nabla(\mathbf{p}\cdot\mathbf{E})$, $\boldsymbol{\tau} = \mathbf{p}\times\mathbf{E}$, $U = -(\mathbf{p}\cdot\mathbf{E})$ $\mathbf{m} = I A\hat{\mathbf{n}} = \frac{1}{2}\int_V \mathbf{r}\times\mathbf{j}(\mathbf{r})dV$ $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{3(\mathbf{m}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}}{r^3}$, $\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\hat{\mathbf{r}}}{r^2}$, $\mathbf{F} = \vec{\nabla}(\mathbf{m}\cdot\mathbf{B})$, $\boldsymbol{\tau} = \mathbf{m}\times\mathbf{B}$
Boundary conditions:	$(\mathbf{E}_2 - \mathbf{E}_1)\cdot\hat{\mathbf{n}}_2 = \frac{\sigma}{\varepsilon_0}$, $(\mathbf{E}_2 - \mathbf{E}_1)\cdot\hat{\mathbf{t}} = 0$, $(\mathbf{D}_2 - \mathbf{D}_1)\cdot\hat{\mathbf{n}}_2 = \sigma_f$ $(\mathbf{B}_2 - \mathbf{B}_1)\cdot\hat{\mathbf{n}}_2 = 0$, $(\mathbf{B}_2 - \mathbf{B}_1)\cdot\hat{\mathbf{t}} = \mu_0\mathbf{k}\cdot\hat{\mathbf{n}}$, $(\mathbf{H}_2 - \mathbf{H}_1)\cdot\hat{\mathbf{t}} = \mathbf{k}_f\cdot\hat{\mathbf{n}}$  Note: $\mathbf{t}\times\mathbf{n}_2 = -\mathbf{n}$
Multipole expansion:	$\phi(r) = \left[\frac{1}{4\pi\varepsilon_0}\right]_{SI}\left[\frac{1}{r}\int_{V'}\rho(\mathbf{r}')dV' + \frac{1}{r^2}\hat{\mathbf{r}}\cdot\int_{V'}\rho(\mathbf{r}')\mathbf{r}'dV'\right.$ $\left. + \frac{1}{r^3}\int_{V'}\frac{3(\hat{\mathbf{r}}\cdot\mathbf{r}')^2 - \mathbf{r}'^2}{2}\rho(\mathbf{r}')dV' + \cdots\right]$ $= \left[\frac{1}{4\pi\varepsilon_0}\right]_{SI}\left[\frac{Q}{r} + \frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2} + \frac{1}{2r^5}\sum_{ij}3x_ix_jQ_{ij} + \cdots\right]$ $Q_{ij} = \int\left(x_i'x_j' - \frac{1}{3}\delta_{ij}r'^2\right)\rho(\mathbf{r}')dV'$, $Q_{ji} = Q_{ij}$, $\sum_i Q_{ii} = 0$ $W = q\phi(0) - \mathbf{p}\cdot\mathbf{E}(0) - \frac{1}{2}\sum_i\sum_j Q_{ij}\frac{\partial E_j}{\partial x_i}\Big\vert_{x_i=0} + \cdots$
Method of images:	Grounded conducting sphere: place $q' = -q\frac{R}{d}$ at $d' = \frac{R^2}{d}$ to make the sphere an equipotential with $\phi = 0$.

Boundary value problems:	$\phi(r,\theta)=\sum_{n=0}^{\infty}(A_n r^n + B_n r^{-(n+1)})P_n(\cos\theta)$
Dielectrics:	$\rho=\rho_f+\rho_p,\quad \sigma=\sigma_f+\sigma_p,\quad \rho_p=-\vec{\nabla}\cdot\boldsymbol{P},\quad \sigma_p=\boldsymbol{P}\cdot\hat{\boldsymbol{n}}\quad \boldsymbol{D}=\varepsilon_0\boldsymbol{E}+\boldsymbol{P}$ lih dielectrics: $\boldsymbol{P}=\varepsilon_0\chi_e\boldsymbol{E}$, $\boldsymbol{D}=\varepsilon_0(1+\chi_e)\boldsymbol{E}=\varepsilon_0 k_e\boldsymbol{E}=\varepsilon\boldsymbol{E}$, $\vec{\nabla}\cdot\boldsymbol{D}=\rho_f,\quad \nabla^2\phi=-\frac{\rho_f}{\varepsilon}$
Magnetic materials:	$\boldsymbol{j}=\boldsymbol{j}_f+\boldsymbol{j}_m,\quad \boldsymbol{k}=\boldsymbol{k}_f+\boldsymbol{k}_m,\quad \boldsymbol{j}_m=\vec{\nabla}\times\boldsymbol{M},\quad \boldsymbol{k}_m=\boldsymbol{M}\times\hat{\boldsymbol{n}},$ $\mathbf{H}=\mathbf{B}/\mu_0-\mathbf{M}$ lih magnetic materials: $\boldsymbol{M}=\chi_m\boldsymbol{H}$, $\boldsymbol{B}=\mu_0(\boldsymbol{H}+\boldsymbol{M})=\mu_0(1+\chi_m)\boldsymbol{H}=\mu_0 k_m\boldsymbol{H}=\mu\boldsymbol{H}$, $\vec{\nabla}\times\boldsymbol{H}=\boldsymbol{j}_f$
Quasi-static situations:	$\varepsilon_i=-\sum_j M_{ij}\frac{\partial I_j}{\partial t}\qquad \varepsilon=-L\frac{\partial I}{\partial t}\qquad U=\frac{1}{2}\sum_{m=1}^{N}F_m I_m,$ $U=\frac{1}{2\mu_0}\int_{all\_space}\boldsymbol{B}\cdot\boldsymbol{B}dV$
Electrodynamics:	$\vec{\nabla}\times\left(\boldsymbol{E}+\frac{\partial\boldsymbol{A}}{\partial t}\right)=0\ \Rightarrow\ \boldsymbol{E}+\frac{\partial\boldsymbol{A}}{\partial t}=-\vec{\nabla}\phi$
Poynting's theorem:	$\boldsymbol{E}\cdot\boldsymbol{j}=-\frac{\partial u}{\partial t}-\vec{\nabla}\cdot\boldsymbol{S}\qquad u=\frac{1}{2\mu_0}B^2+\frac{\varepsilon_0}{2}E^2\qquad \boldsymbol{S}=\frac{1}{\mu_0}\boldsymbol{E}\times\boldsymbol{B}$
The Lorentz gauge:	$\vec{\nabla}\cdot\boldsymbol{A}=-\frac{1}{c^2}\frac{\partial\phi}{\partial t}$
The wave equation:	$\left(\nabla^2-\mu\varepsilon\frac{\partial^2}{\partial t^2}\right)\boldsymbol{E}=0\qquad \left(\nabla^2-\mu\varepsilon\frac{\partial^2}{\partial t^2}\right)\boldsymbol{B}=0$ $\left(\nabla^2-\mu\varepsilon\frac{\partial^2}{\partial t^2}-\mu\sigma_c\frac{\partial}{\partial t}\right)\boldsymbol{E}=0,\qquad \left(\nabla^2-\mu\varepsilon\frac{\partial^2}{\partial t^2}-\mu\sigma_c\frac{\partial}{\partial t}\right)\boldsymbol{B}=0,$ $k^2=i\mu\sigma_c\omega+\mu\varepsilon\omega^2=\mu\varepsilon(\omega)\omega^2$
Electromagnetic radiation:	$\boldsymbol{E}(\boldsymbol{r},t)=-\left[\frac{1}{4\pi\varepsilon_0}\right]_{SI}\frac{q}{c^2r''}\boldsymbol{a}_\perp(t-\frac{r''}{c}),\qquad \boldsymbol{r}''=\boldsymbol{r}-\boldsymbol{r}'(t-\frac{\lvert\boldsymbol{r}-\boldsymbol{r}'\rvert}{c}),$ $\boldsymbol{B}=\frac{\hat{\boldsymbol{r}}''}{c}\times\boldsymbol{E}$
Larmor formula:	$P=\oint\vec{S}\cdot\hat{n}dA=\frac{2}{3}\frac{e^2a^2}{c^3}=q^2a^2/(6\pi\varepsilon_0c^3)$
4-vectors:	$j^\mu=(c\rho,\mathbf{j})$ = 4-vector current, $A^\mu=(\phi/c,\mathbf{A})$ = 4-vector potential
Transformation of the fields:	$\mathbf{E}'_\parallel=\mathbf{E}_\parallel$, $\mathbf{B}'_\parallel=\mathbf{B}_\parallel$, $\mathbf{E}'_\perp=\gamma(\mathbf{E}+\mathbf{v}\times\mathbf{B})_\perp$, $\mathbf{B}'_\perp=\gamma(\mathbf{B}-(\mathbf{v}/c^2)\times\mathbf{E})_\perp$
Lorentz invariants:	$\mathbf{E}^2-c^2\mathbf{B}^2$, $(\mathbf{E}\cdot\mathbf{B})^2$

Quantum Mechanics

WKB approximation:	$\oint$ pdx = $\oint$ ηkdx = (n + 1/2)h, $k^2 = (2m/\eta^2)(E - V(x))$, V(x) finite everywhere. In regions where E > V(x) we have $\phi(x) = Ak^{-1/2}\exp(\pm i\int^x k(x')dx')$, and in regions where E < V(x) we have $\phi(x) = A\rho^{-1/2}\exp(\pm\int^x \rho k(x')dx')$.
Harmonic oscillator:	$\phi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} \exp\left(-\frac{1}{2}\frac{m\omega x^2}{\hbar}\right)$ $\phi_1(x) = \left(\frac{4}{\pi}\left(\frac{m\omega}{\hbar}\right)^3\right)^{\frac{1}{4}} x\exp\left(-\frac{1}{2}\frac{m\omega x^2}{\hbar}\right)$ $\phi_2(x) = \left(\frac{m\omega}{4\pi\hbar}\right)^{\frac{1}{4}} \left[2\frac{m\omega}{\hbar}x^2 - 1\right] \exp\left(-\frac{1}{2}\frac{m\omega x^2}{\hbar}\right)$ $\phi_n(x) = \frac{1}{\sqrt{n!}}\frac{1}{\sqrt{2^n}}\left(\frac{\beta}{\sqrt{\pi}}\right)^{\frac{1}{2}} H_n(\eta)\exp\left(-\frac{1}{2}\eta^2\right)$, where $\eta = \sqrt{\frac{m\omega}{\hbar}}\, x = \beta x$
Angular momentum:	$[J_i,J_j] = \varepsilon_{ijk}i\eta J_k$, $[J_i,J^2] = 0$, $J^2\|k,j,m\rangle = j(j+1)\eta^2\|k,j,m\rangle$, $J_z\|k,j,m\rangle = m\eta\|k,j,m\rangle$, $J_+ = J_x + iJ_y$, $J_+ = J_x - iJ_y$, $J_\pm\|k,j,m\rangle = [j(j+1)-m(m\pm1)]^{1/2}\eta\|k,j,m\pm1\rangle$.
Orbital angular momentum:	$L^2 = -\hbar^2\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right)$ $\qquad L_z = \frac{\hbar}{i}\frac{\partial}{\partial\phi}$ $Y_{00} = \frac{1}{\sqrt{4\pi}}$, $\quad Y_{1\pm1}(\theta,\phi) = \mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\phi}$, $\quad Y_{10}(\theta,\phi) = \sqrt{\frac{3}{4\pi}}\cos\theta$ $Y_{2\pm2} = \sqrt{\frac{15}{32\pi}}\sin^2\theta e^{\pm 2i\phi}$, $Y_{2\pm1}(\theta,\phi) = \mp\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta e^{\pm i\phi}$, $Y_{20}(\theta,\phi) = \sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$
Spin 1/2:	$(S_z) = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\qquad (S_x) = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $(S_y) = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\qquad (S^2) = \frac{3\hbar^2}{4}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.
Particle in a central potential:	$H = -(\eta^2/(2m))(1/r)(\partial^2/\partial r^2)r + L^2/(2mr^2) + V(r)$,

	$\psi_{klm}(r,\theta,\phi) = R_{kl}(r)Y_{lm}(\theta,\phi) = [u_{kl}(r)/r]Y_{lm}(\theta,\phi)$, $[-(\eta^2/(2m))(\partial^2/\partial r^2) + \eta^2 l(l+1)/(2mr^2) + V(r)]u_{kl}(r) = E_{kl}u_{kl}(r)$.
Stationary perturbation theory:	$E_1^p = <\phi_p \vert W \vert \phi_p>$, $\vert \psi_p^1> = \Sigma_{p'\neq p,i}\, b_{p'}{}^i \vert \phi_{p'}{}^i >$, where $b_{p'}{}^i = <\phi_{p'}{}^i \vert W \vert \phi_p> / (E_0{}^p - E_0{}^{p'})$, $E_2^p = \Sigma_{p'\neq p,i} \vert <\phi_{p'}{}^i \vert W \vert \phi_p> \vert ^2 / (E_0{}^p - E_0{}^{p'})$.
Time-dependent perturbation theory: Fermi's golden rule:	$P_{if}(t) = (1/\eta^2)\vert\int_0^t \exp(i\omega_{fi}t')W_{fi}(t')dt'\vert^2$ Let $W(t) = W\exp(\pm i\omega t)$, then $w(i,\beta E) = (2\pi/\eta)\rho(\beta,E)\vert W_{Ei}\vert^2\delta_{E-Ei,\eta\omega}$, where $W_{Ei} = <\phi_E\vert W\vert\phi_i>$.
Scattering:	Asymptotic form: $\phi_k(r) = \exp(ikz) + f_k(\theta)\exp(ikr)/r$. $f_k(\theta) = (1/k)\Sigma_{l=0}{}^{\infty}(2l+1)\exp(i\delta_l)\sin\delta_l P_l(\cos\theta)$, $d\sigma_k/d\Omega = \vert f_k(\theta)\vert^2 = (1/k^2)\vert\Sigma_{l=0}{}^{\infty}(2l+1)\exp(i\delta_l)\sin\delta_l P_l(\cos\theta)\vert^2$. $\sigma_k{}^B(\theta,\phi) = \sigma_k{}^B(\mathbf{k},\mathbf{k}') = [\mu^2/(4\pi^2\eta^4)]\vert\int d^3r' \exp(-i\mathbf{q}\cdot\mathbf{r}')V(\mathbf{r}')\vert^2$, where $\mathbf{q} = \mathbf{k} - \mathbf{k}'$, $\mathbf{k} = \mu\mathbf{v}_0/\eta$, $\mathbf{k}' = \mu v_0/\eta$ $(\mathbf{k}'/k')$, and is the reduced mass.

Vector identities:

Note: **A**, **B**, **C**, and **D** are vectors.

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})$$

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\cdot\mathbf{C})\mathbf{B} - (\mathbf{A}\cdot\mathbf{B})\mathbf{C}$$

$$(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D}) = (\mathbf{A}\cdot\mathbf{C})(\mathbf{B}\cdot\mathbf{D}) - (\mathbf{A}\cdot\mathbf{D})(\mathbf{B}\cdot\mathbf{C})$$

Note: **A** and **B** are vector fields, ψ and ϕ are scalar fields.

$$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$$

$$\nabla\cdot(\psi\mathbf{A}) = \mathbf{A}\cdot\nabla\psi + \psi\nabla\cdot\mathbf{A}$$

$$\nabla\times(\psi\mathbf{A}) = \psi\nabla\times\mathbf{A} - \mathbf{A}\times\nabla\psi$$

$$\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$$

$$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}(\nabla\times\mathbf{B})$$

$$\nabla\times(\mathbf{A}\times\mathbf{B}) = \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A}) + (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B}$$

$$(\mathbf{A}\cdot\nabla)\mathbf{A} = \frac{1}{2}\nabla|\mathbf{A}|^2 + (\nabla\times\mathbf{A})\times\mathbf{A}$$

$$\nabla\times\nabla\phi = 0$$

$$\nabla\cdot(\nabla\times\mathbf{A}) = 0$$

$$\nabla\cdot\nabla\phi = \nabla^2\phi$$

$$\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$$

Integral relations:

$\int_V (\nabla\cdot\mathbf{F})dV = \oint_S \mathbf{F}\cdot d\mathbf{a}$	Divergence theorem
$\int_S (\nabla\times\mathbf{F})\cdot d\mathbf{a} = \oint_C \mathbf{F}\cdot d\mathbf{s}$	Stokes' theorem
$\int_S \phi(\nabla\psi)\cdot d\mathbf{a} = \int_V [\phi\nabla^2\psi + (\nabla\phi)\cdot(\nabla\psi)]\,dV.$	Green's first identity
$\int_V (\phi\nabla^2\psi - \psi\nabla^2\phi)\,dV = \int_S (\phi\nabla\psi - \psi\nabla\phi)\cdot d\mathbf{a}.$	Green's second identity

Gradient, divergence, curl, and Laplacian:

Note: **A** is a vector fields and *f* is a scalar fields.

Cartesian coordinates:

∇f	=	$\frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$
$\nabla \cdot \mathbf{A}$	=	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$
$\nabla \times \mathbf{A}$	=	$(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z})\hat{\mathbf{x}} \quad +$ $(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x})\hat{\mathbf{y}} \quad +$ $(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y})\hat{\mathbf{z}}$
$\nabla^2 f$	=	$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

Cylindrical coordinates:

∇f	=	$\frac{\partial f}{\partial \rho}\hat{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}}$
$\nabla \cdot \mathbf{A}$	=	$\frac{1}{\rho}\frac{\partial \rho A_\rho}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$
$\nabla \times \mathbf{A}$	=	$(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z})\hat{\boldsymbol{\rho}} \quad +$ $(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho})\hat{\boldsymbol{\phi}} \quad +$ $\frac{1}{\rho}(\frac{\partial \rho A_\phi}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi})\hat{\boldsymbol{z}}$
$\nabla^2 f$	=	$\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho\frac{\partial f}{\partial \rho}) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$

Spherical coordinates:

∇f	=	$\frac{\partial f}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\phi}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}}$
$\nabla \cdot \mathbf{A}$	=	$\frac{1}{r^2}\frac{\partial r^2 A_r}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial A_\theta \sin\theta}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$
$\nabla \times \mathbf{A}$	=	$\frac{1}{r\sin\theta}(\frac{\partial A_\phi \sin\theta}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi})\hat{\boldsymbol{r}} \quad +$ $(\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{1}{r}\frac{\partial r A_\phi}{\partial r})\hat{\boldsymbol{\theta}} \quad +$ $\frac{1}{r}(\frac{\partial r A_\theta}{\partial r} - \frac{\partial A_r}{\partial \theta})\hat{\boldsymbol{\phi}}$
$\nabla^2 f$	=	$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial f}{\partial r}) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\frac{\partial f}{\partial \theta}) + \frac{1}{r^2\sin\theta}\frac{\partial^2 f}{\partial \phi^2}$