

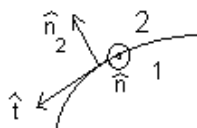
Formulas:

Mechanics

Motion in a non-inertial frame:	$m d\mathbf{v}/dt = -\partial U/\partial \mathbf{r} - m d\mathbf{V}/dt + m \mathbf{r} \times d\boldsymbol{\Omega}/dt - 2m\boldsymbol{\Omega} \times \mathbf{v} - m\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})$
Lagrange's equations:	$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} = 0, \quad \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} = Q_j,$ $Q_j = -\frac{\partial U}{\partial q_j} \quad \text{or} \quad Q_j = -\frac{\partial U}{\partial q_j} + \frac{d}{dt} \left(\frac{\partial U}{\partial \dot{q}_j} \right)$
Hamilton's equations:	$\dot{q}_i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H}{\partial q_i}, \quad H(q, p, t) = \sum_i p_i \dot{q}_i - L, \quad \frac{\partial L}{\partial \dot{q}_j} = p_j$
Lagrange multipliers:	$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_k} \right) - \frac{\partial L}{\partial q_k} = \sum_l \lambda_l a_{lk}, \quad \sum_k a_{lk} dq_k + a_{lt} dt = 0, \quad l = 1, \dots, m$
Small oscillations:	$L = \frac{1}{2} \sum_{ij} \left(T_{ij} \dot{q}_i \dot{q}_j - k_{ij} q_i q_j \right) \quad \text{with} \quad T_{ij} = T_{ji}, \quad k_{ij} = k_{ji}$ $\sum_j \left(k_{ij} - \omega_a^2 T_{ij} \right) A_{ja} = 0$ $q_j = \text{Re} \sum_a \left(C_a A_{ja} e^{i\omega_a t} \right)$
Motion in a central potential:	$m\ddot{r} - \frac{M^2}{mr^3} = f(r), \quad \text{or} \quad m\ddot{r} = -\frac{\partial U_{\text{eff}}(r)}{\partial r}, \quad \text{with} \quad f(r) = -\frac{\partial U(r)}{\partial r}$ $\frac{M^2 u^2}{m} \left(\frac{d^2 u}{d\phi^2} + u \right) = -f(u)$ $r = \pm \sqrt{\frac{2}{m} \left(E - U_{\text{eff}}(r) \right)}, \quad \phi = \phi_0 - \int \frac{M du}{\sqrt{2m \left(E - U_{\text{eff}}(u) \right)}}$ <p>Kepler orbit: $\frac{p}{r} = 1 + e \cos(\phi - \phi_0)$</p>
Two interacting particles:	<p>CM frame:</p> $L = \frac{1}{2} \mu \dot{\mathbf{r}}^2 - U(r), \quad \mu = \frac{m_1 m_2}{m_1 + m_2}, \quad \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$

Relativistic kinematics:	$\begin{pmatrix} x'_0 \\ x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x'_0 \\ x'_1 \\ x'_2 \\ x'_3 \end{pmatrix}$
4-vectors:	$(u_0, \mathbf{u}) = \left(\gamma c, \gamma \frac{d\mathbf{r}}{dt} \right) = \left(\frac{dx_0}{d\tau}, \frac{d\mathbf{r}}{d\tau} \right), \quad (u_0, \mathbf{u}) \cdot (u_0, \mathbf{u}) = c^2$ $(p_0, \mathbf{p}) = (mu_0, m\mathbf{u}) = \left(\gamma mc, \gamma m \frac{d\mathbf{r}}{dt} \right), \quad p_0 = \frac{E}{c} = \gamma mc, \quad \mathbf{p} = \gamma m \frac{d\mathbf{r}}{dt}$ $(p_0, \mathbf{p}) \cdot (p_0, \mathbf{p}) = m^2 c^2 \Rightarrow E^2 = m^2 c^4 + p^2 c^2$
Transformation of velocities:	$u_{\parallel} = (u_{\parallel} - v)/(1 - \mathbf{v} \cdot \mathbf{u}/c^2), \quad u_{\perp} = u_{\perp}/(\gamma(1 - \mathbf{v} \cdot \mathbf{u}/c^2))$
Doppler shift:	$\omega' = \gamma\omega(1 - (v/c)\cos\theta)$
Relativistic collisions:	<p>For each component p_{μ} of the 4-vector (p_0, p_1, p_2, p_3) we have</p> $\sum_{\text{particles_in}} p_{\mu} = \sum_{\text{particles_out}} p_{\mu}, \quad \text{or} \quad \sum_i (p_i)_{\mu} = \sum_j (p_j)_{\mu}.$ <p>For transformations between reference frames we have</p> $(P_0, \mathbf{P}) \cdot (P_0, \mathbf{P}) = (P'_0, \mathbf{P}') \cdot (P'_0, \mathbf{P}'),$ <p>where $P_0 = \sum_{\text{particles}} p_0$ and $\mathbf{P} = \sum_{\text{particles}} \mathbf{p}$.</p>

E&M

<p>Maxwell's equations:</p>	$\vec{\nabla} \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}, \quad \vec{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \vec{\nabla} \cdot \mathbf{B} = 0, \quad \vec{\nabla} \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$ $\vec{\nabla} \cdot \mathbf{D} = \rho_f, \quad \vec{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \vec{\nabla} \cdot \mathbf{B} = 0, \quad \vec{\nabla} \times \mathbf{H} = \mathbf{j}_f + \frac{\partial \mathbf{D}}{\partial t}$ <p>Linear materials:</p> $\vec{\nabla} \cdot \mathbf{E} = \frac{\rho_f}{\epsilon}, \quad \vec{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ $\vec{\nabla} \cdot \mathbf{B} = 0, \quad \vec{\nabla} \times \mathbf{B} = \mu \mathbf{j}_f + \mu \epsilon \frac{\partial \mathbf{E}}{\partial t}$
<p>Dipole field and potential:</p>	$\mathbf{E}(\mathbf{r}) = \left[\frac{1}{4\pi\epsilon_0} \right]_{SI} \frac{1}{r^3} \left[3(\mathbf{p} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} - \mathbf{p} \right] \quad \phi(\mathbf{r}) = \left[\frac{1}{4\pi\epsilon_0} \right]_{SI} \frac{\mathbf{p} \cdot \hat{\mathbf{r}}}{r^2}$ <p>$\mathbf{F} = \nabla(\mathbf{p} \cdot \mathbf{E}), \quad \boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}, \quad U = -(\mathbf{p} \cdot \mathbf{E})$</p> $\mathbf{m} = I A \hat{\mathbf{n}} = \frac{1}{2} \int_V \mathbf{r} \times \mathbf{j}(\mathbf{r}) dV$ $\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{3(\mathbf{m} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} - \mathbf{m}}{r^3}, \quad \mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \hat{\mathbf{r}}}{r^2},$ <p>$\mathbf{F} = \vec{\nabla}(\mathbf{m} \cdot \mathbf{B}), \quad \boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}$</p>
<p>Boundary conditions:</p>	$(\mathbf{E}_2 - \mathbf{E}_1) \cdot \hat{\mathbf{n}}_2 = \frac{\sigma}{\epsilon_0}, \quad (\mathbf{E}_2 - \mathbf{E}_1) \cdot \hat{\mathbf{t}} = 0, \quad (\mathbf{D}_2 - \mathbf{D}_1) \cdot \hat{\mathbf{n}}_2 = \sigma_f$ $(\mathbf{B}_2 - \mathbf{B}_1) \cdot \hat{\mathbf{n}}_2 = 0, \quad (\mathbf{B}_2 - \mathbf{B}_1) \cdot \hat{\mathbf{t}} = \mu_0 \mathbf{k} \cdot \hat{\mathbf{n}}, \quad (\mathbf{H}_2 - \mathbf{H}_1) \cdot \hat{\mathbf{t}} = \mathbf{k}_f \cdot \hat{\mathbf{n}}$  <p>Note: $\mathbf{t} \times \mathbf{n}_2 = -\mathbf{n}$</p>
<p>Multipole expansion:</p>	$\phi(\mathbf{r}) = \left[\frac{1}{4\pi\epsilon_0} \right]_{SI} \left[\frac{1}{r} \int_V \rho(\mathbf{r}') dV' + \frac{1}{r^2} \hat{\mathbf{r}} \cdot \int_V \rho(\mathbf{r}') \mathbf{r}' dV' + \frac{1}{r^3} \int_V \frac{3(\hat{\mathbf{r}} \cdot \mathbf{r}')^2 - r'^2}{2} \rho(\mathbf{r}') dV' + \dots \right]$ $= \left[\frac{1}{4\pi\epsilon_0} \right]_{SI} \left[\frac{Q}{r} + \frac{\mathbf{p} \cdot \hat{\mathbf{r}}}{r^2} + \frac{1}{2r^3} \sum_{ij} 3x_i x_j Q_{ij} + \dots \right]$ $Q_{ij} = \int \left(x'_i x'_j - \frac{1}{3} \delta_{ij} r'^2 \right) \rho(\mathbf{r}') dV', \quad Q_{ii} = Q_{jj}, \quad \sum_i Q_{ii} = 0$ $W = q\phi(0) - \mathbf{p} \cdot \mathbf{E}(0) - \frac{1}{2} \sum_i \sum_j Q_{ij} \frac{\partial E_j}{\partial x_i} \Big _{x_i=0} + \dots$
<p>Method of images:</p>	<p>Grounded conducting sphere: place $q' = -q \frac{R}{d}$ at $d' = \frac{R^2}{d}$ to make the sphere an equipotential with $\phi = 0$.</p>

Boundary value problems:	$\phi(r, \theta) = \sum_{n=0}^{\infty} (A_n r^n + B_n r^{-(n+1)}) P_n(\cos \theta)$
Dielectrics:	$\rho = \rho_f + \rho_p, \quad \sigma = \sigma_f + \sigma_p, \quad \rho_p = -\vec{\nabla} \cdot \mathbf{P}, \quad \sigma_p = \mathbf{P} \cdot \hat{n} \quad \mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$ <p>lih dielectrics: $\mathbf{P} = \epsilon_0 \chi_e \mathbf{E}$, $\mathbf{D} = \epsilon_0 (1 + \chi_e) \mathbf{E} = \epsilon_0 k_e \mathbf{E} = \epsilon \mathbf{E}$,</p> $\vec{\nabla} \cdot \mathbf{D} = \rho_f, \quad \nabla^2 \phi = -\frac{\rho_f}{\epsilon}$
Magnetic materials:	$\mathbf{j} = \mathbf{j}_f + \mathbf{j}_m, \quad \mathbf{k} = \mathbf{k}_f + \mathbf{k}_m, \quad \mathbf{j}_m = \vec{\nabla} \times \mathbf{M}, \quad \mathbf{k}_m = \mathbf{M} \times \hat{n},$ $\mathbf{H} = \mathbf{B}/\mu_0 - \mathbf{M}$ <p>lih magnetic materials: $\mathbf{M} = \chi_m \mathbf{H}$,</p> $\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) = \mu_0 (1 + \chi_m) \mathbf{H} = \mu_0 k_m \mathbf{H} = \mu \mathbf{H} , \quad \vec{\nabla} \times \mathbf{H} = \mathbf{j}_f$
Quasi-static situations:	$\epsilon_i = -\sum_j M_{ij} \frac{\partial I_j}{\partial t} \quad \epsilon = -L \frac{\partial I}{\partial t} \quad U = \frac{1}{2} \sum_{m=1}^N F_m I_m,$ $U = \frac{1}{2\mu_0} \int_{\text{all space}} \mathbf{B} \cdot \mathbf{B} dV$
Electrodynamics:	$\vec{\nabla} \times \left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} \right) = 0 \Rightarrow \mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\vec{\nabla} \phi$
Poynting's theorem:	$\mathbf{E} \cdot \mathbf{j} = -\frac{\partial u}{\partial t} - \vec{\nabla} \cdot \mathbf{S} \quad u = \frac{1}{2\mu_0} B^2 + \frac{\epsilon_0}{2} E^2 \quad \mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$
The Lorentz gauge:	$\vec{\nabla} \cdot \mathbf{A} = -\frac{1}{c^2} \frac{\partial \phi}{\partial t}$
The wave equation:	$\left(\nabla^2 - \mu\epsilon \frac{\partial^2}{\partial t^2} \right) \mathbf{E} = 0 \quad \left(\nabla^2 - \mu\epsilon \frac{\partial^2}{\partial t^2} \right) \mathbf{B} = 0$ $\left(\nabla^2 - \mu\epsilon \frac{\partial^2}{\partial t^2} - \mu\sigma_c \frac{\partial}{\partial t} \right) \mathbf{E} = 0, \quad \left(\nabla^2 - \mu\epsilon \frac{\partial^2}{\partial t^2} - \mu\sigma_c \frac{\partial}{\partial t} \right) \mathbf{B} = 0,$ $k^2 = i\mu\sigma_c\omega + \mu\epsilon\omega^2 = \mu\epsilon(\omega)\omega^2$
Electromagnetic radiation:	$\mathbf{E}(\mathbf{r}, t) = -\left[\frac{1}{4\pi\epsilon_0} \right]_{SI} \frac{q}{c^2 r''} \mathbf{a}_{\perp}(t - \frac{r''}{c}), \quad \mathbf{r}'' = \mathbf{r} - \mathbf{r}'(t - \frac{ \mathbf{r} - \mathbf{r}' }{c}),$ $\mathbf{B} = \frac{\hat{\mathbf{r}}''}{c} \times \mathbf{E}$
Larmor formula:	$P = \oint \vec{S} \cdot \hat{n} dA = \frac{2}{3} \frac{e^2 a^2}{c^3} = q^2 a^2 / (6\pi\epsilon_0 c^3)$
4-vectors:	$\mathbf{j}^\mu = (c\rho, \mathbf{j}) = 4\text{-vector current}, \quad \mathbf{A}^\mu = (\phi/c, \mathbf{A}) = 4\text{-vector potential}$
Transformation of the fields:	$\mathbf{E}'_{\parallel} = \mathbf{E}_{\parallel}, \quad \mathbf{B}'_{\parallel} = \mathbf{B}_{\parallel} = 0, \quad \mathbf{E}'_{\perp} = \gamma(\mathbf{E} + \mathbf{v} \times \mathbf{B})_{\perp}, \quad \mathbf{B}'_{\perp} = \gamma(\mathbf{B} - (\mathbf{v}/c^2) \times \mathbf{E})_{\perp}$
Lorentz invariants:	$\mathbf{E}^2 - c^2 \mathbf{B}^2, (\mathbf{E} \cdot \mathbf{B})^2$

Quantum Mechanics

WKB approximation:	$\oint p dx = \oint \hbar k dx = (n + 1/2)\hbar, \quad k^2 = (2m/\hbar^2)(E - V(x)),$ $V(x) \text{ finite everywhere.}$ <p>In regions where $E > V(x)$ we have $\phi(x) = A k^{-1/2} \exp(\pm i \int^x k(x') dx'),$</p> <p>and in regions where $E < V(x)$ we have $\phi(x) = A \rho^{-1/2} \exp(\pm \int^x \rho k(x') dx').$</p>
Harmonic oscillator:	$\phi_0(x) = \left(\frac{m\omega}{\pi\hbar} \right)^{\frac{1}{4}} \exp\left(-\frac{1}{2} \frac{m\omega x^2}{\hbar}\right)$ $\phi_1(x) = \left(\frac{4}{\pi} \left(\frac{m\omega}{\hbar} \right)^3 \right)^{\frac{1}{4}} x \exp\left(-\frac{1}{2} \frac{m\omega x^2}{\hbar}\right)$ $\phi_2(x) = \left(\frac{m\omega}{4\pi\hbar} \right)^{\frac{1}{4}} \left[2 \frac{m\omega}{\hbar} x^2 - 1 \right] \exp\left(-\frac{1}{2} \frac{m\omega x^2}{\hbar}\right)$ $\phi_n(x) = \frac{1}{\sqrt{n!}} \frac{1}{\sqrt{2^n}} \left(\frac{\beta}{\sqrt{\pi}} \right)^{\frac{1}{2}} H_n(\eta) \exp\left(-\frac{1}{2} \eta^2\right), \quad \text{where } \eta = \sqrt{\frac{m\omega}{\hbar}} x = \beta x$
Angular momentum:	$[J_i, J_j] = \epsilon_{ijk} i\hbar J_k, \quad [J_i, J^2] = 0, \quad J^2 k, j, m\rangle = j(j+1)\hbar^2 k, j, m\rangle,$ $J_z k, j, m\rangle = m\hbar k, j, m\rangle,$ $J_+ = J_x + iJ_y, \quad J_- = J_x - iJ_y,$ $J_{\pm} k, j, m\rangle = [j(j+1) - m(m \pm 1)]^{1/2} \hbar k, j, m \pm 1\rangle.$
Orbital angular momentum:	$L^2 = -\hbar^2 \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \right) \quad L_z = \frac{\hbar}{i} \frac{\partial}{\partial \phi}$ $Y_{00} = \frac{1}{\sqrt{4\pi}}, \quad Y_{1\pm 1}(\theta, \phi) = \mp \sqrt{\frac{3}{8\pi}} \sin \theta e^{\pm i\phi}, \quad Y_{10}(\theta, \phi) = \sqrt{\frac{3}{4\pi}} \cos \theta$ $Y_{2\pm 2} = \sqrt{\frac{15}{32\pi}} \sin^2 \theta e^{\pm 2i\phi}, \quad Y_{2\pm 1}(\theta, \phi) = \mp \sqrt{\frac{15}{8\pi}} \sin \theta \cos \theta e^{\pm i\phi},$ $Y_{20}(\theta, \phi) = \sqrt{\frac{5}{16\pi}} (3 \cos^2 \theta - 1)$
Spin 1/2:	$(S_x) = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (S_x) = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $(S_y) = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad (S^2) = \frac{3\hbar^2}{4} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$

Particle in a central potential:	$H = -(\hbar^2/(2m))(1/r)(\partial^2/\partial r^2)r + L^2/(2mr^2) + V(r),$ $\psi_{klm}(r,\theta,\phi) = R_{kl}(r)Y_{lm}(\theta,\phi) = [u_{kl}(r)/r]Y_{lm}(\theta,\phi),$ $[-(\hbar^2/(2m))(\partial^2/\partial r^2) + \hbar^2 l(l+1)/(2mr^2) + V(r)]u_{kl}(r) = E_{kl}u_{kl}(r).$
Stationary perturbation theory:	$E_{1P} = \langle \phi_p W \phi_p \rangle, \quad \psi_{p^1}\rangle = \sum_{p' \neq p, i} b_{p^1}^i \phi_{p^i}\rangle,$ <p>where $b_{p^1}^i = \langle \phi_{p^i} W \phi_p \rangle / (E_{0P} - E_{0P^i})$,</p> $E_{2P} = \sum_{p' \neq p, i} \langle \phi_{p^i} W \phi_p \rangle ^2 / (E_{0P} - E_{0P^i}).$
Time-dependent perturbation theory: Fermi's golden rule:	$P_{if}(t) = (1/\hbar^2) \left \int_0^t \exp(i\omega_{fi}t') W_{fi}(t') dt' \right ^2$ <p>Let $W(t) = W \exp(\pm i\omega t)$, then $w(i, \beta E) = (2\pi/\hbar) \rho(\beta, E) W_{Ei} ^2 \delta_{E-Ei, \hbar\omega}$, where $W_{Ei} = \langle \phi_E W \phi_i \rangle$.</p>
Scattering:	<p>Asymptotic form: $\phi_k(r) = \exp(ikz) + f_k(\theta) \exp(ikr)/r$.</p> $f_k(\theta) = (1/k) \sum_{l=0}^{\infty} (2l+1) \exp(i\delta_l) \sin \delta_l P_l(\cos \theta),$ $d\sigma_k/d\Omega = f_k(\theta) ^2 = (1/k^2) \left \sum_{l=0}^{\infty} (2l+1) \exp(i\delta_l) \sin \delta_l P_l(\cos \theta) \right ^2.$ $\sigma_k^B(\theta, \phi) = \sigma_k^B(\mathbf{k}, \mathbf{k}') = [\mu^2/(4\pi^2 \hbar^4)] \left \int d^3r' \exp(-i\mathbf{q} \cdot \mathbf{r}') V(\mathbf{r}') \right ^2,$ <p>where $\mathbf{q} = \mathbf{k} - \mathbf{k}'$, $\mathbf{k} = \mu v_0/\hbar$, $\mathbf{k}' = \mu v_0/\hbar (\mathbf{k}'/k')$, and is the reduced mass.</p>

Vector identities:

Note: \mathbf{A} , \mathbf{B} , \mathbf{C} , and \mathbf{D} are vectors.

$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C}$$

$$(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C})$$

Note: \mathbf{A} and \mathbf{B} are vector fields, ψ and ϕ are scalar fields.

$$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi$$

$$\nabla \cdot (\psi\mathbf{A}) = \mathbf{A} \cdot \nabla\psi + \psi\nabla \cdot \mathbf{A}$$

$$\nabla \times (\psi\mathbf{A}) = \psi\nabla \times \mathbf{A} - \mathbf{A} \times \nabla\psi$$

$$\nabla(\mathbf{A} \cdot \mathbf{B}) = \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A}$$

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B})$$

$$\nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A}(\nabla \cdot \mathbf{B}) - \mathbf{B}(\nabla \cdot \mathbf{A}) + (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B}$$

$$(\mathbf{A} \cdot \nabla)\mathbf{A} = \frac{1}{2}\nabla|\mathbf{A}|^2 + (\nabla \times \mathbf{A}) \times \mathbf{A}$$

$$\nabla \times \nabla\phi = \mathbf{0}$$

$$\nabla \cdot (\nabla \times \mathbf{A}) = 0$$

$$\nabla \cdot \nabla\phi = \nabla^2\phi$$

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A}$$

Integral relations:

$$\int_V (\nabla \cdot \mathbf{F})dV = \oint_S \mathbf{F} \cdot d\mathbf{a} \quad \text{Divergence theorem}$$

$$\int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{a} = \oint_C \mathbf{F} \cdot d\mathbf{s} \quad \text{Stokes' theorem}$$

$$\int_S \phi(\nabla\psi) \cdot d\mathbf{a} = \int_V [\phi\nabla^2\psi + (\nabla\phi) \cdot (\nabla\psi)] dV. \quad \text{Green's first identity}$$

$$\int_V (\phi\nabla^2\psi - \psi\nabla^2\phi) dV = \int_S (\phi\nabla\psi - \psi\nabla\phi) \cdot d\mathbf{a}. \quad \text{Green's second identity}$$

Gradient, divergence, curl, and Laplacian:

Note: \mathbf{A} is a vector fields and f is a scalar fields.

Cartesian coordinates:

∇f	=	$\frac{\partial f}{\partial x} \hat{\mathbf{x}} + \frac{\partial f}{\partial y} \hat{\mathbf{y}} + \frac{\partial f}{\partial z} \hat{\mathbf{z}}$
$\nabla \cdot \mathbf{A}$	=	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$
$\nabla \times \mathbf{A}$	=	$(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}) \hat{\mathbf{x}} +$ $(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}) \hat{\mathbf{y}} +$ $(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}) \hat{\mathbf{z}}$
$\nabla^2 f$	=	$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

Cylindrical coordinates:

∇f	=	$\frac{\partial f}{\partial \rho} \hat{\boldsymbol{\rho}} + \frac{1}{\rho} \frac{\partial f}{\partial \phi} \hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z} \hat{\mathbf{z}}$
$\nabla \cdot \mathbf{A}$	=	$\frac{1}{\rho} \frac{\partial \rho A_\rho}{\partial \rho} + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$
$\nabla \times \mathbf{A}$	=	$(\frac{1}{\rho} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}) \hat{\boldsymbol{\rho}} +$ $(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}) \hat{\boldsymbol{\phi}} +$ $\frac{1}{\rho} (\frac{\partial \rho A_\phi}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi}) \hat{\mathbf{z}}$
$\nabla^2 f$	=	$\frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho \frac{\partial f}{\partial \rho}) + \frac{1}{\rho^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$

Spherical coordinates:

∇f	=	$\frac{\partial f}{\partial r} \hat{\mathbf{r}} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\boldsymbol{\theta}} + \frac{1}{r \sin \phi} \frac{\partial f}{\partial \phi} \hat{\boldsymbol{\phi}}$
$\nabla \cdot \mathbf{A}$	=	$\frac{1}{r^2} \frac{\partial r^2 A_r}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial A_\theta \sin \theta}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$
$\nabla \times \mathbf{A}$	=	$\frac{1}{r \sin \theta} (\frac{\partial A_\phi \sin \theta}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}) \hat{\mathbf{r}} +$ $(\frac{1}{r \sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{1}{r} \frac{\partial r A_\phi}{\partial r}) \hat{\boldsymbol{\theta}} +$ $\frac{1}{r} (\frac{\partial r A_\theta}{\partial r} - \frac{\partial A_r}{\partial \theta}) \hat{\boldsymbol{\phi}}$
$\nabla^2 f$	=	$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial f}{\partial r}) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \frac{\partial f}{\partial \theta}) + \frac{1}{r^2 \sin \theta} \frac{\partial^2 f}{\partial \phi^2}$