

Main points:

In Physics 513 we covered relativistic kinematics and dynamics. We practiced the Lorentz transformation of 4-vectors and specifically considered 4 different 4-vectors.

$$\begin{aligned} \text{4-vector position:} & \quad x^\mu = (x_0, \mathbf{r}) \\ \text{4-vector velocity:} & \quad u^\mu = (\gamma c, \gamma \mathbf{v}) \\ \text{4-vector momentum:} & \quad p^\mu = (\gamma mc, \gamma m\mathbf{v}) = (E/c, \mathbf{p}) \\ \text{4-vector wave vector:} & \quad k^\mu = (\omega/c, \mathbf{k}) \end{aligned}$$

The dot product of any two 4-vectors,  $(a_0, \mathbf{a}) \cdot (b_0, \mathbf{b}) = a_0 b_0 - \mathbf{a} \cdot \mathbf{b}$  is Lorentz invariant.

In relativistic E&M we encounter some additional 4-vectors.

$$\begin{aligned} \text{4-vector current:} & \quad j^\mu = (c\rho, \mathbf{j}) \\ \text{4-vector potential:} & \quad A^\mu = (\Phi/c, \mathbf{A}) \end{aligned}$$

Note:  $j^\mu$  and  $A^\mu$  are in general function of position and time. For the transformed expressions to make sense in frame  $K'$ , we also need to transform the position 4-vector, so that we have  $j'^\mu(x'^\mu)$  and  $A'^\mu(x'^\mu)$ .

We also show that vector operators can transform as 4-vectors.

The electric and magnetic fields are NOT components of a 4-vector but of a tensor.

For the exam you need to be able to use the following transformation properties,  
Assume reference frame  $K'$  moves with velocity  $\mathbf{v}$  with respect to reference frame  $K$ .

$$\begin{aligned} \mathbf{E}'_{\parallel} &= \mathbf{E}_{\parallel}, \quad \mathbf{B}'_{\parallel} = \mathbf{B}_{\parallel}, \\ \mathbf{E}'_{\perp} &= \gamma(\mathbf{E} + \mathbf{v} \times \mathbf{B})_{\perp}, \\ \mathbf{B}'_{\perp} &= \gamma(\mathbf{B} - (\mathbf{v}/c^2) \times \mathbf{E})_{\perp}. \end{aligned}$$

Here  $\parallel$  and  $\perp$  refer to the direction of the relative velocity.

For the fields, we also have scalar quantities that is invariant under a Lorentz transformation.

$E^2 - c^2 B^2$  and  $(\mathbf{E} \cdot \mathbf{B})^2$  are invariant under a Lorentz transformation.

To calculate the electromagnetic force on a charged particle in two different reference frame we discuss two approaches.

- (i) Transform the fields and use  $\mathbf{F}' = q(\mathbf{E}' + \mathbf{v} \times \mathbf{B}')$ .
- (ii) Transform the momentum 4-vector and use  $F' = d\mathbf{p}'/dt'$ .

We review the Doppler shift formula and show that we can compare intensities by transforming the fields.